

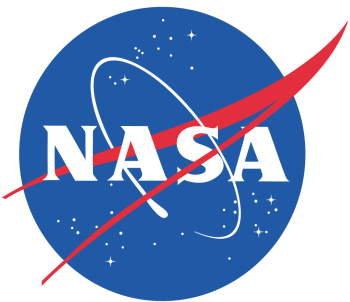
# Data-driven Predictive Modeling of Turbulent Flows : Current Status and Challenges

Karthik Duraisamy



Thanks to:

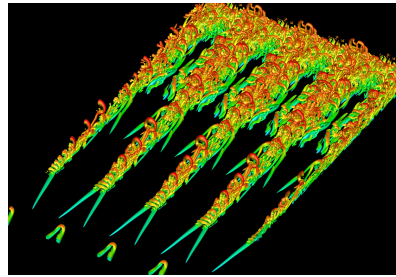
A.P. Singh, B. Tracey, S. Medida, J. Alonso, P. Durbin



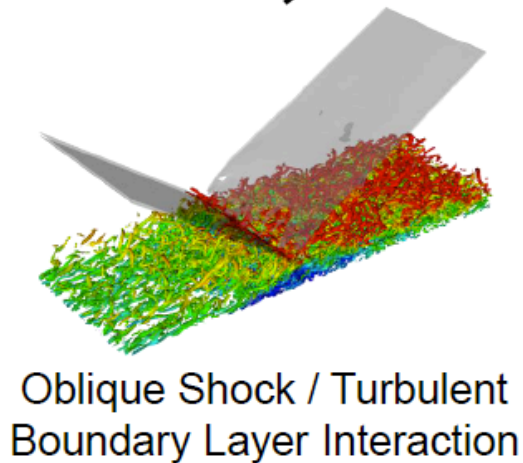
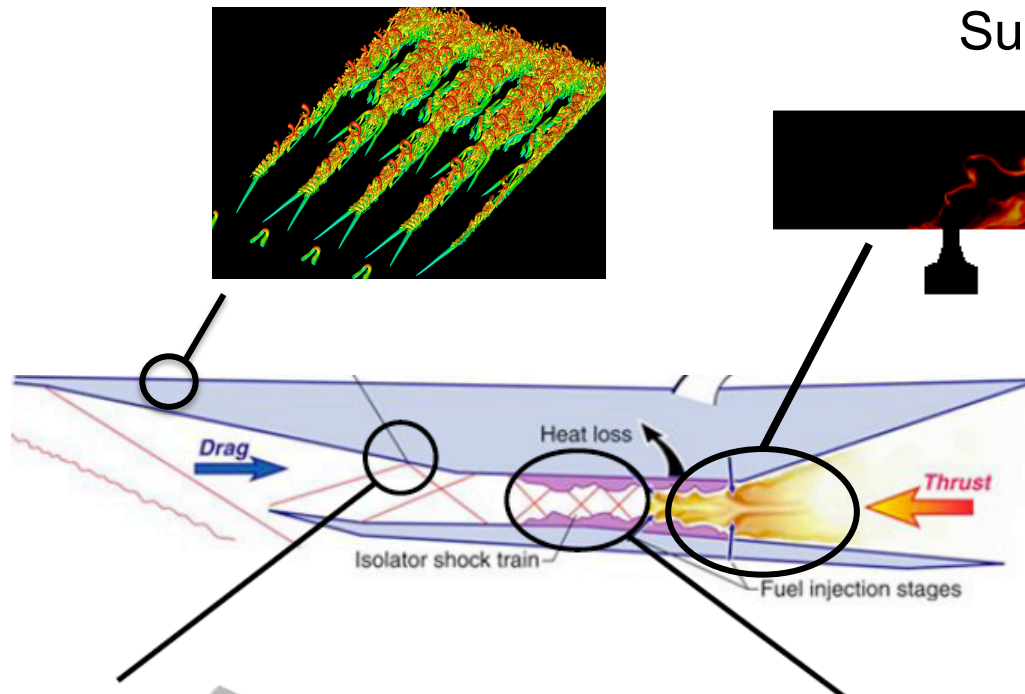
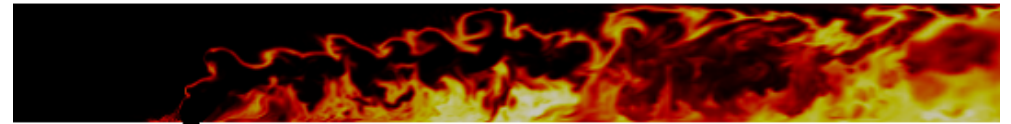
Aug 17 2016, Langley

# Turbulence : a “tyranny of scales”

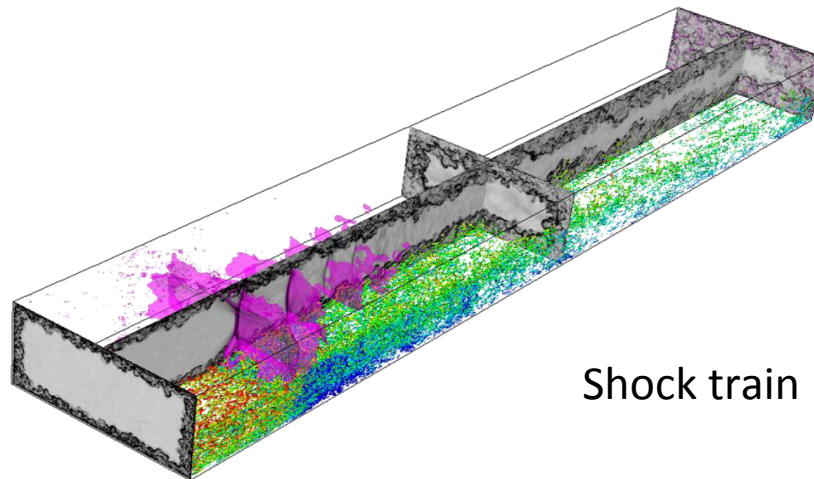
Transitional boundary layer



Supersonic combustion



Oblique Shock / Turbulent  
Boundary Layer Interaction



Shock train

# Turbulence Modeling: Coarse-graining the Navier-Stokes equations

NSE  
(DNS)

$$\rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial u_i u_j}{\partial x_j} = \rho f_i + \frac{\partial}{\partial x_j} [-p + 2\mu S_{ij}]$$

Decompose

$$u_i = \bar{u}_i + u'_i \quad ; \quad p = \bar{p} + p'$$

Averaged  
(RANS)

$$\rho \frac{\partial \bar{u}_i}{\partial t} + \rho \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = \rho \bar{f}_i + \frac{\partial}{\partial x_j} \left[ -\bar{p} + 2\mu \bar{S}_{ij} - \overline{\rho u'_i u'_j} \right]$$

The closure is

$$\tau_{ij}^{RANS} = -\overline{\rho u'_i u'_j}$$

# Modeling the Reynolds stress tensor

The diagram illustrates the decomposition of the Reynolds stress tensor into exact and approximate components. The word "Exact" is written in green at the top, with three blue arrows pointing to the terms  $C_{ij}$ ,  $P_{ij}$ , and  $V_{ij}$  in the equation. The word "Approx" is written in red at the bottom, with four blue arrows pointing to the terms  $T_{ij}^m$ ,  $\Pi_{ij}^m$ ,  $K_{ij}^m$ , and  $D_{ij}^m$  in the equation.

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} = C_{ij} + P_{ij} + V_{ij} + T_{ij}^m + \Pi_{ij}^m + K_{ij}^m + D_{ij}^m$$

- Models found lacking in accuracy in many complex flows
- It is the balance between the terms that matters (and not accuracy of individual terms)
  - ➔ Still respect invariance, symmetries, etc.
- Many “seemingly physical” quantities are just operational variables
  - ➔ Use of apriori analysis is of limited utility
- Start with clean idea, but loss of rigor in final model
- Model constants calibrated on very limited data



# Our approach

We propose large-scale data-driven to enable the construction of accurate models of turbulence

- ➔ Focus on non-parametric (functional) improvements
- ➔ Not replacing existing modeling knowledge, but just building on it

This technique enables

- ➔ The ability to “infer” what’s missing in the closure
- ➔ The ability to convert that inference into modeling knowledge

# A timeline of data in turbulence modeling

## UQ perspective

Moser et al (2011-2013)	: Bayesian analysis and model averaging (coefficients)
Wang et al (2012)	: <b>Infer</b> correction to eddy viscosity coefficient
Tracey, Duraisamy, Alonso (2012)	: <b>Machine Learning</b> of Reynolds stress perturbations
Edeling (2014)	: <b>Infer</b> uncertainty in model coefficients
Ling et al (2015)	: <b>Machine Learning</b> to determine regions of model error
Xiao et al (2015)	: <b>Infer</b> reynolds stress perturbations

## Modeling perspective

Duraisamy et al. (2014)	: <b>Inversion</b> and <b>machine learning</b> for Turbulence modeling
Tracey, Duraisamy, Alonso (2015)	: <b>Machine learning</b> + <b>embedding</b>
Xiao et al (2016)	: <b>Inference</b> + <b>learning</b> of Reynolds stress perturbations
Duraisamy (2016)	: <b>Inference</b> + <b>machine learning</b> + <b>embedding</b>

# Perturbing eddy viscosity

## UQ perspective

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Duraisamy et al. (2014)	: Inversion and machine learning for Turbulence modeling
Tracey, Duraisamy, Alonso (2015)	: Machine learning + embedding
Xiao et al (2016)	: Inference + learning of Reynolds stress perturbations
Duraisamy (2016)	: Inference + machine learning + embedding

$$R_p = 2(\mu_t + \delta) S_{ij} - \frac{2}{3} \rho k \delta_{ij}$$

# Perturbing Reynolds stress tensor

## UQ perspective

Moser et al (2011-2013)	: Bayesian analysis and model averaging (coefficients)
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$$\mathbf{R}_p = 2k \left[ \frac{\mathbf{I}}{3} + \mathbf{V}(\Lambda + \delta_\Lambda) \mathbf{V}^T \right]$$

Structure  
proposed by  
Emory &  
Iaccarino (2012)

# Introduce corrective terms in the model

## UQ perspective

Moser et al (2011-2013)	: Bayesian analysis and model averaging (coefficients)
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$$\frac{DQ}{Dt} = \mathcal{R}(Q) + \delta(Q) \rightarrow \frac{DQ}{Dt} = \beta(Q)\mathcal{R}(Q) \rightarrow \beta \equiv 1 + \frac{\delta}{R}$$

# Outline

- Proof-of-concept
  - ➔ If there is a known underlying model, can we discover it?
- The general framework
  - ➔ How do we setup the data-driven turbulence modeling problem
- Demonstration
  - ➔ Predictions in Airfoil flows
- Computer Science, Scaling, etc...

# Proof-of-concept

- Basic questions: Can machine learning work at all?:
  - Can a learning algorithm discover and replicate a known model?
  - Will the learned model destabilize a PDE solver?
- Isolate errors in learning from complexities of real-world data

Not just a matter of learning and prediction... Have to address convergence within framework

# Proof-of-concept : Replicating Spalart Allmaras Model

$$\mu_t = \rho \hat{\nu} f_{v1}$$

$$\underbrace{\frac{\partial \hat{\nu}}{\partial t} + u_j \frac{\partial \hat{\nu}}{\partial x_j}}_{\text{Convection}} = \underbrace{c_{b1}(1-f_{t2})\hat{S}\hat{\nu}}_{\text{Production}} - \underbrace{\left(c_{w1}f_w - \frac{c_{b1}}{\kappa^2}f_{t2}\right)\left(\frac{\hat{\nu}}{d}\right)^2}_{\text{Destruction}} + \underbrace{\frac{1}{\sigma}\left(\frac{\partial}{\partial x_j}\left((\nu+\hat{\nu})\frac{\partial \hat{\nu}}{\partial x_j}\right)\right)}_{\text{Diffusion}} + \underbrace{c_{b2}\frac{\partial \hat{\nu}}{\partial x_i}\frac{\partial \hat{\nu}}{\partial x_i}}_{\text{Cross Production}}$$

$$\chi = \hat{\nu}/\nu$$

$$f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}$$

$$f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}}$$

$$\hat{S} = \Omega + \frac{\hat{\nu}}{\kappa^2 d^2} f_{v2}$$

$$r = \min\left[\frac{\hat{\nu}}{\hat{S}\kappa^2 d^2}, 10\right]$$

$$g = r + c_{w2}(r^6 - r)$$

$$f_w = g \left[ \frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right]^{1/6}$$

$$f_{t2} = c_{t3} \exp(-c_{t4}\chi^2)$$

$$W_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$

$$\Omega = \sqrt{2W_{ij}W_{ij}}$$



# Proof-of-concept : Replicating Spalart Allmaras Model

$$\underbrace{\frac{\partial \hat{\nu}}{\partial t} + u_j \frac{\partial \hat{\nu}}{\partial x_j}}_{\text{Convection}} = \underbrace{\hspace{10em}}_{\text{Production}} - \underbrace{\hspace{10em}}_{\text{Destruction}} + \underbrace{\frac{1}{\sigma} \left( \frac{\partial}{\partial x_j} \left( (\nu + \hat{\nu}) \frac{\partial \hat{\nu}}{\partial x_j} \right) \right)}_{\text{Diffusion}} + \underbrace{\hspace{10em}}_{\text{Cross Production}}$$

Locally Non-Dimensional  
Input Features

$$\chi = \hat{\nu} / \nu$$

$$\bar{\Omega} = \frac{d^2}{\hat{\nu} + \nu} \Omega$$

$$\bar{N} = \frac{d^2}{(\hat{\nu} + \nu)^2} N$$

Locally Non-Dimensional  
Outputs

$$s_p = c_{b1}(1 - f_{t2})\hat{S}\hat{\nu}$$

$$s_d = \left( c_{w1}f_w - \frac{c_{b1}}{\kappa^2}f_{t2} \right) \left( \frac{\hat{\nu}}{d} \right)^2$$

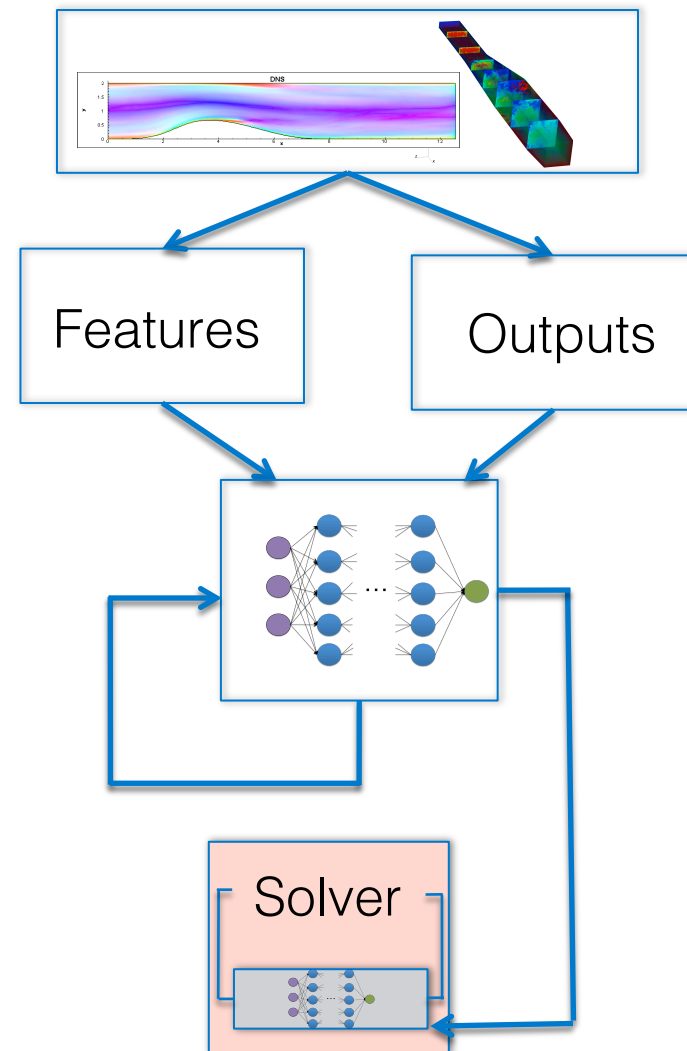
$$s_{cp} = \frac{c_{b2}}{\sigma} \frac{\partial \hat{\nu}}{\partial x_i} \frac{\partial \hat{\nu}}{\partial x_i}$$

$$s = s_p + s_d + s_{cp}$$

$$\bar{s}_i = \left( \frac{d}{\hat{\nu}} \right)^2 s_i$$

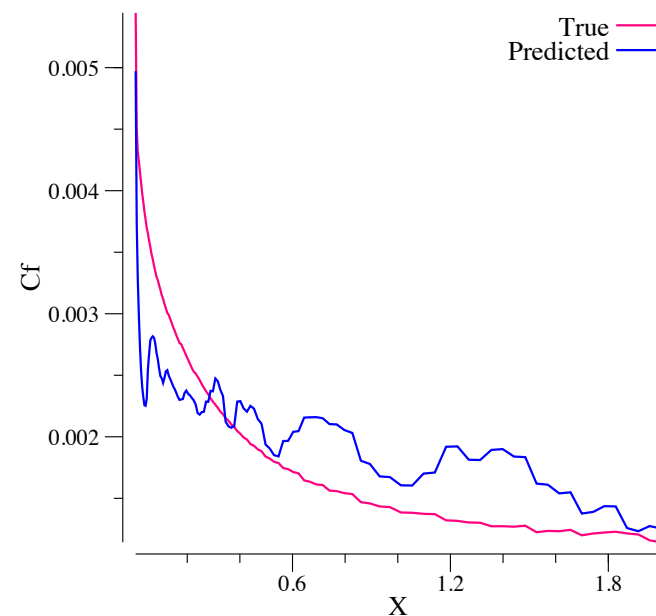
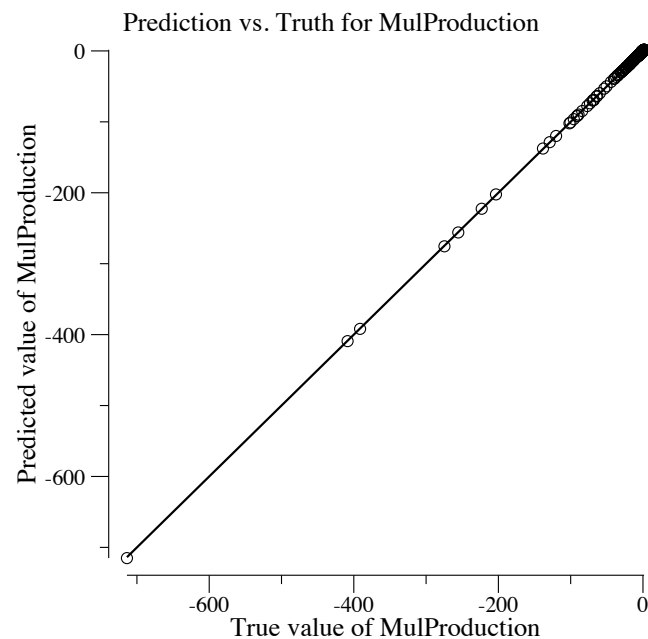
# Procedure

- 1) Select representative datasets
  - Flat plates, pressure-driven channels, airfoils
- 2) Choose and extract input and output features
  - Spalart-Allmaras quantities
- 3) Select learning algorithm
  - Neural network
- 4) Train learning algorithm
  - BFGS optimizer
- 5) Embed learned model within flow solver
  - SU2



# We can learn and we can test, but ...

- Favorable pressure gradient channel flow



- Injection within a converging solver yields poor results

# The loss function

- ▶ Squared-Error

$$L = \sum_{i=1}^k (p_i - t_i)^2$$

- ▶ Penalizes differences in the output value

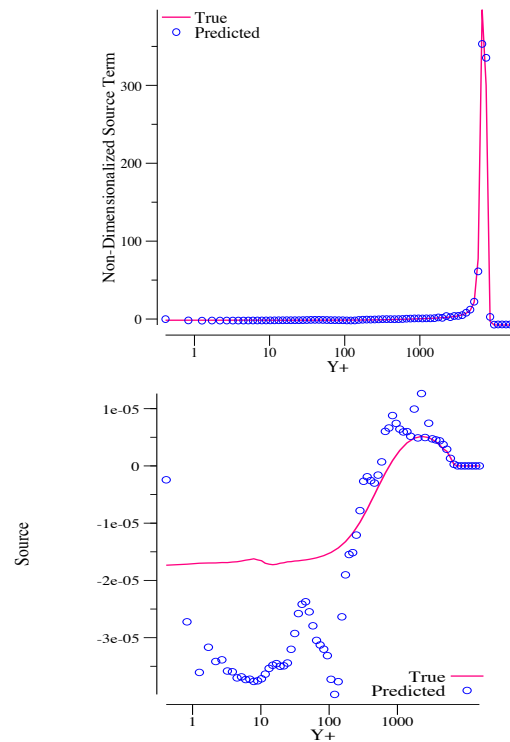
- ▶ Dimensionalized Squared-Error

$$L_2 = \sum_{i=1}^k \left( \left( \frac{d_i^2}{(\hat{\nu}_i + \nu_i)^2} \right) p_{\bar{s},i} - t_{s,i} \right)^2$$

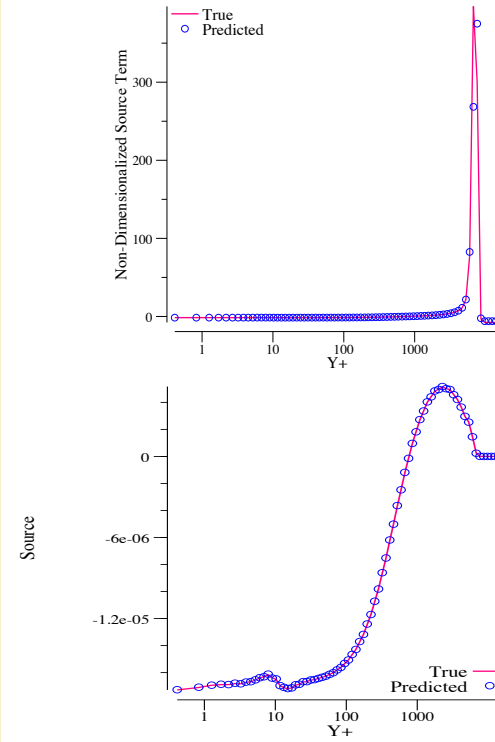
- ▶ Penalizes differences in the dimensional output value

# The loss function

## Squared-distance Loss



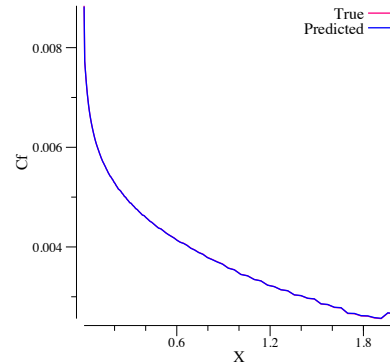
## Dimensionalized Loss



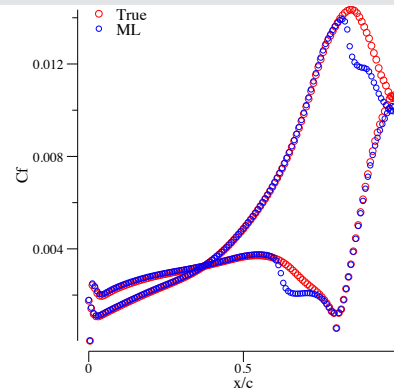
➤ Must align loss function with CFD environment

# Test cases

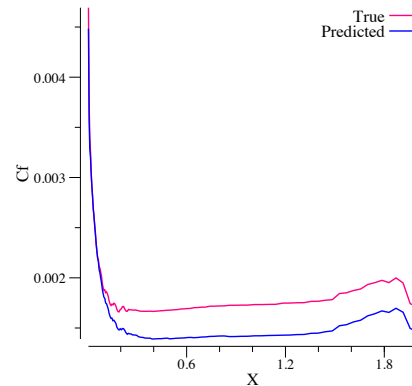
➤ G: No major difference



➤ F: Small region of difference



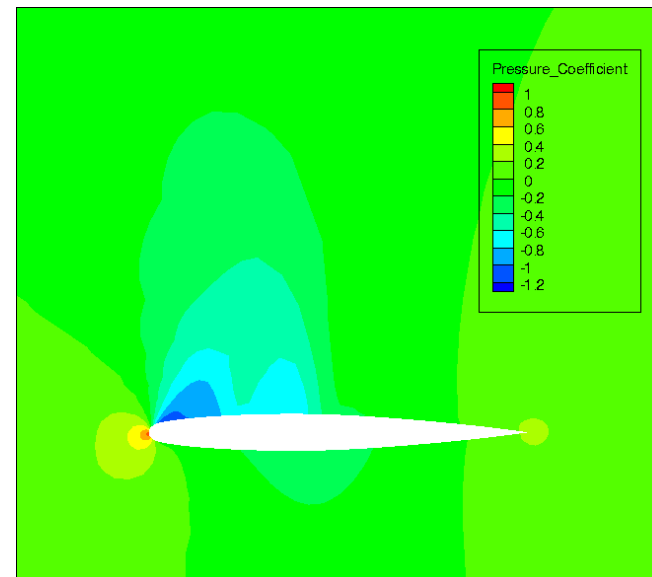
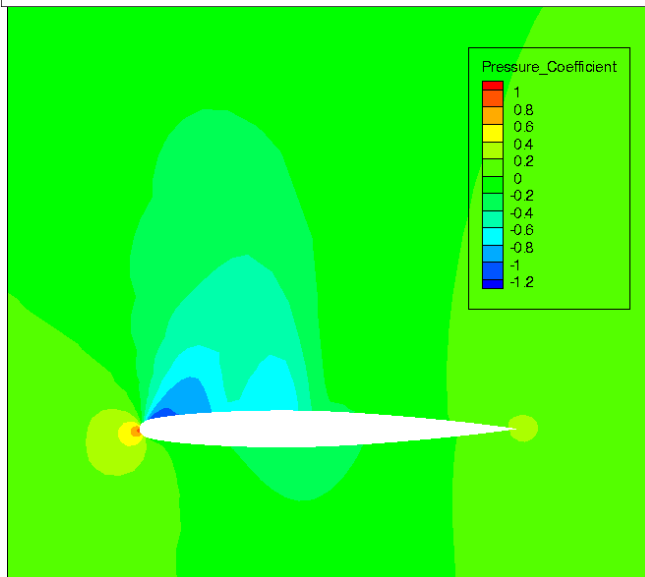
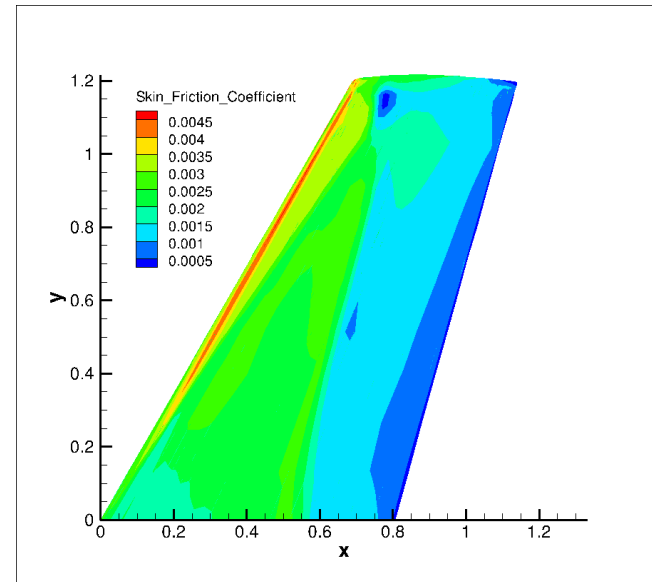
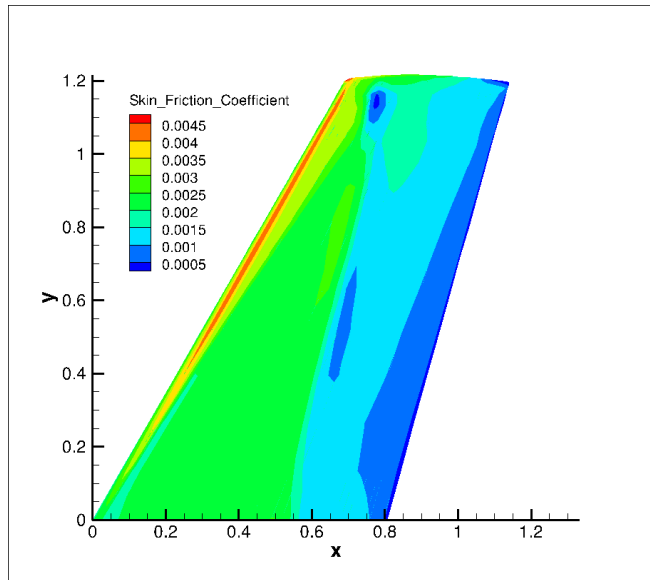
➤ P: Large discrepancy



	Dest.	$F_w$	Mul. Dest.	Mul. Prod.	Prod.	Source
<b>Flatplate 3e6</b>	G	G	G	G	G	G
Flatplate 4e6	G	G	G	G	G	G
<b>Flatplate 5e6</b>	G	G	G	G	G	G
Flatplate 6e6	G	G	G	G	G	G
<b>Flatplate 7e6</b>	G	G	G	G	G	G
Channel $C_p = -0.3$	G	G	G	G	G	F
Channel $C_p = -0.1$	G	G	G	G	G	F
Channel $C_p = -0.03$	G	G	G	G	G	F
Channel $C_p = -0.01$	G	G	G	G	G	F
Channel $C_p = 0.01$	G	G	G	G	G	F
Channel $C_p = 0.03$	G	G	G	G	G	F
Channel $C_p = 0.1$	G	G	G	G	G	F
Channel $C_p = 0.3$	P	G	G	G	P	F
NACA 0	G	G	G	G	G	G
NACA 1	G	G	G	G	G	G
NACA 2	G	G	G	G	G	G
NACA 3	G	G	G	G	G	G
NACA 4	G	G	G	G	G	G
NACA 5	G	G	G	G	G	G
NACA 6	G	G	G	G	G	G
NACA 7	G	G	G	G	G	G
NACA 8	G	G	G	F	G	G
NACA 9	G	G	G	F	G	G
NACA 10	G	G	G	F	G	G
NACA 11	G	G	G	F	G	G
NACA 12	G	G	G	F	G	G

450+ cases

# Test on 3D problem



True

ML

# Takeaways - 1

- ✦ Feature Scaling is important
- ✦ Testing within the CFD solver
- ✦ Alignment of loss function

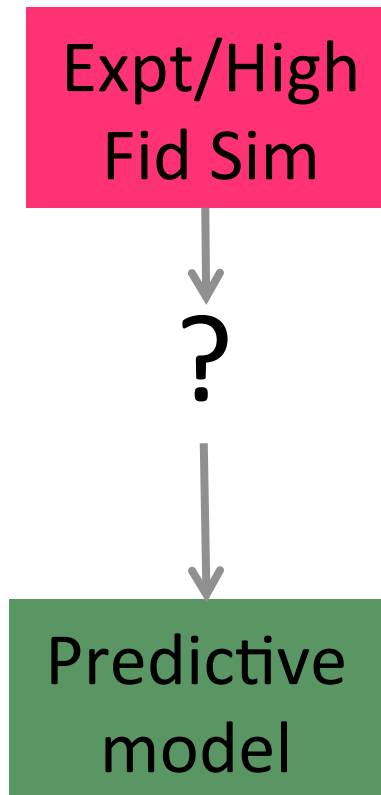
IF THERE IS AN UNDERLYING MODEL, IT IS POSSIBLE  
TO DISCOVER IT



# Outline

- Proof-of-concept
  - ➔ If there is a known underlying model, can we discover it?
- The general framework
  - ➔ How do we setup the data-driven turbulence modeling problem
- Demonstration
  - ➔ Predictions in Airfoil flows
- Computer Science, Scaling, etc...

## On to the real problem



- Data is not available in a form or context that is immediately useful,
- Right data isn't available
- Generalizing specific information into modeling knowledge is hard
- Uncertainties abound
- **NO proof that there is an underlying model waiting to be discovered**

# Field Inversion & Machine learning (FIML)

Datasets  $Y^1, Y^2 \dots Y^n$

Field  
Inversion

$$\frac{DQ}{Dt} = R(Q) + \delta^j(x) : \min_{\delta^j(x)} \|Y^j - Y^j(Q)\|$$

Information Spatial discrepancy



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Information Spatial discrepancy

$$\delta^1(x), \delta^2(x), \dots \delta^n(x)$$

Machine  
Learning

Knowledge Functional discrepancy

$$\hat{\delta}(f(Q))$$

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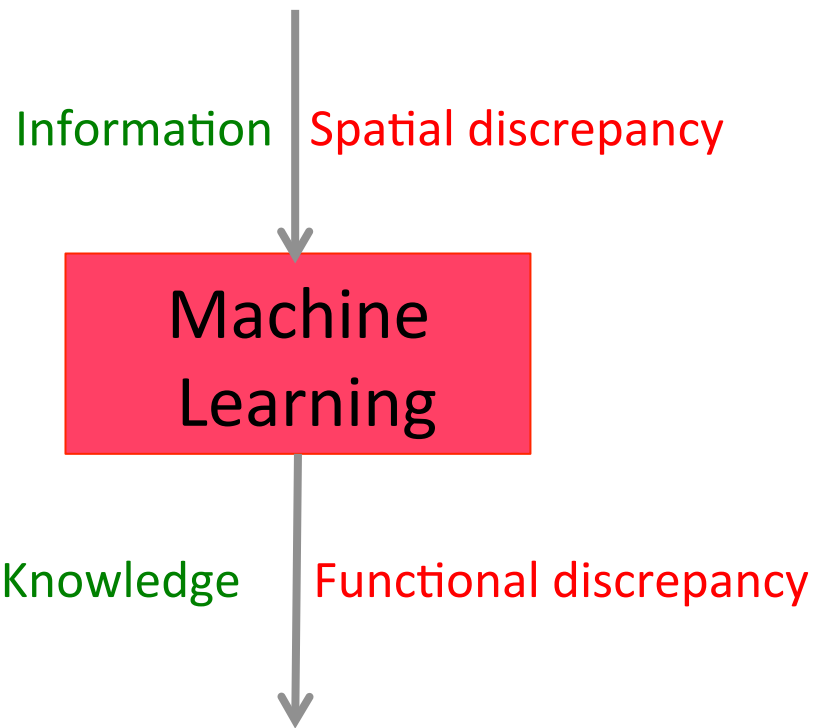
$$\hat{\delta}(f(Q))$$

Embedding

$$\frac{DQ}{Dt} = R(Q) + \hat{\delta}(f(Q))$$

Prediction : Injection into solver

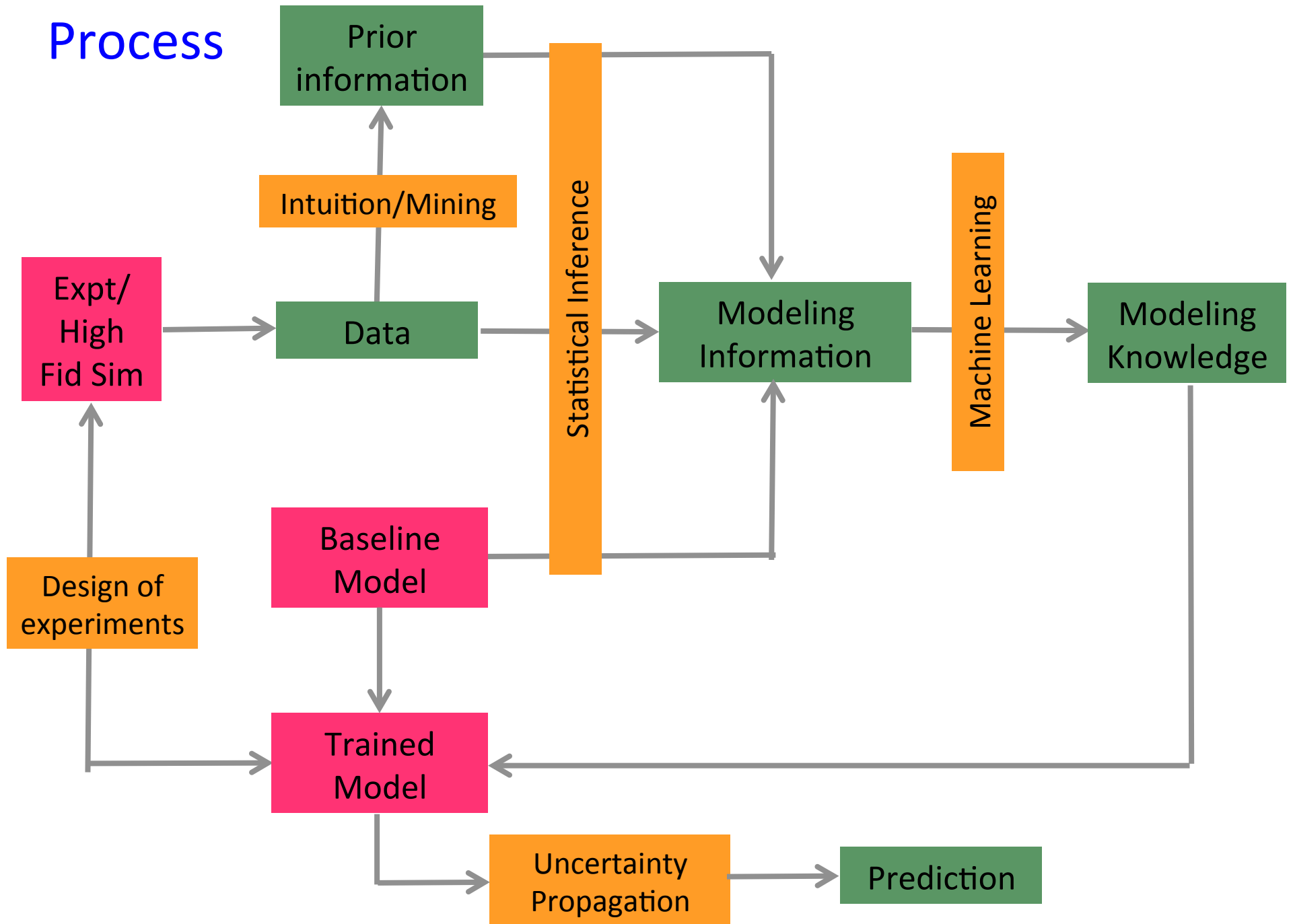
# Major insight from NASA LEARN project



$$\delta^1(x), \delta^2(x), \dots \delta^n(x)$$

$$\hat{\delta}(f(Q))$$

# Process





1) Inference

3) Machine  
Learning

2) Design of  
Experiments

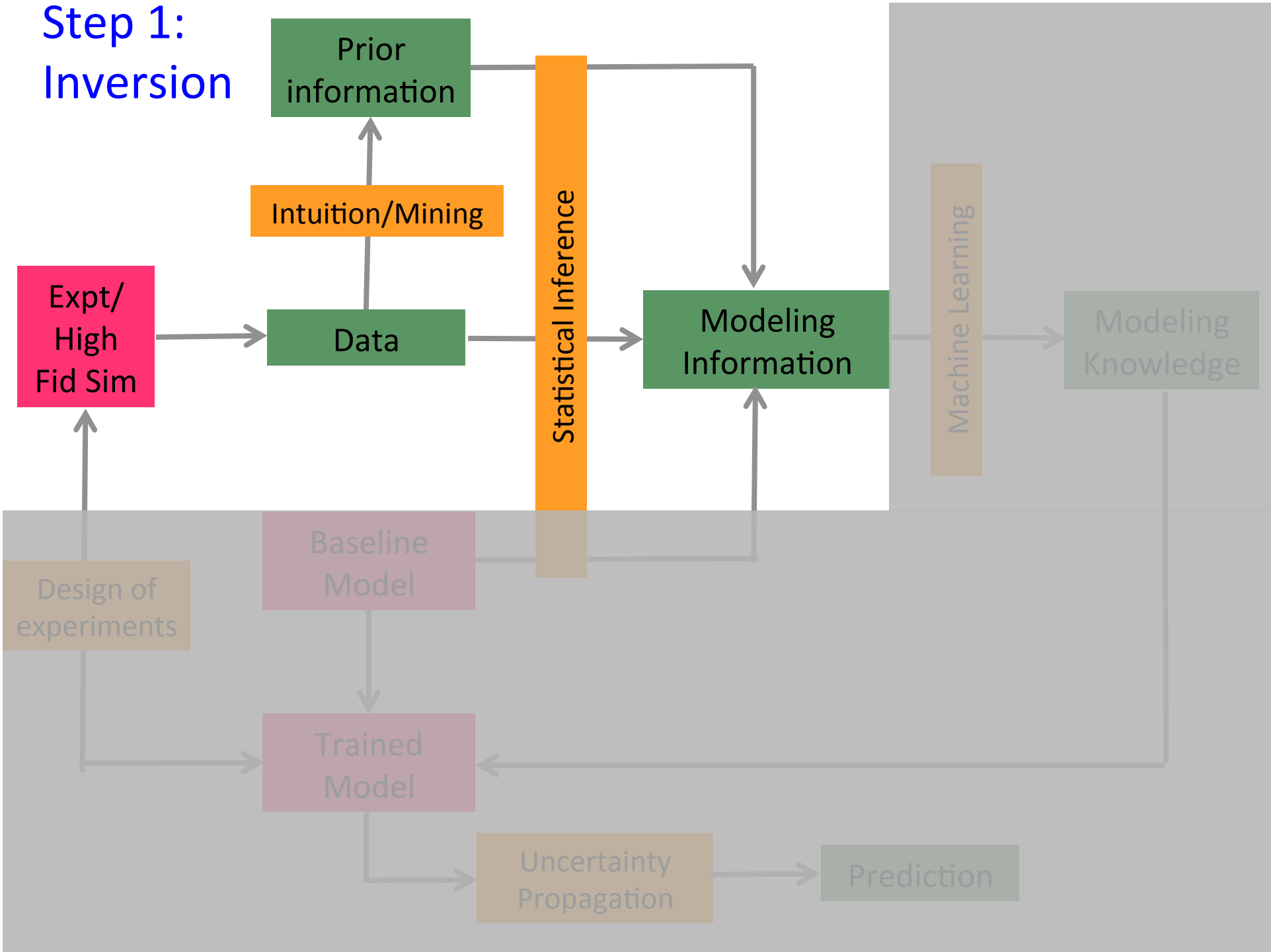
4) Prediction



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The diagram illustrates the 'Step 1: Inversion' process, which is divided into two main sections by a horizontal line. The top section, labeled 'Step 1: Inversion' in blue text, shows the flow from 'Expt/ High Fid Sim' (pink box) to 'Data' (green box), then to 'Modeling Information' (green box). 'Data' also feeds into 'Intuition/Mining' (orange box), which leads to 'Prior information' (green box). 'Prior information' and 'Modeling Information' are connected by a vertical orange bar labeled 'Statistical Inference'. 'Modeling Information' is also connected to 'Machine Learning' (brown box), which leads to 'Modeling Knowledge' (grey box). The bottom section, which is faded, shows the flow from 'Design of experiments' (brown box) to 'Expt/ High Fid Sim'. 'Baseline Model' (pink box) leads to 'Trained Model' (pink box). 'Trained Model' leads to 'Uncertainty Propagation' (brown box), which leads to 'Prediction' (grey box). 'Modeling Knowledge' also feeds into 'Trained Model'.



## Original Model

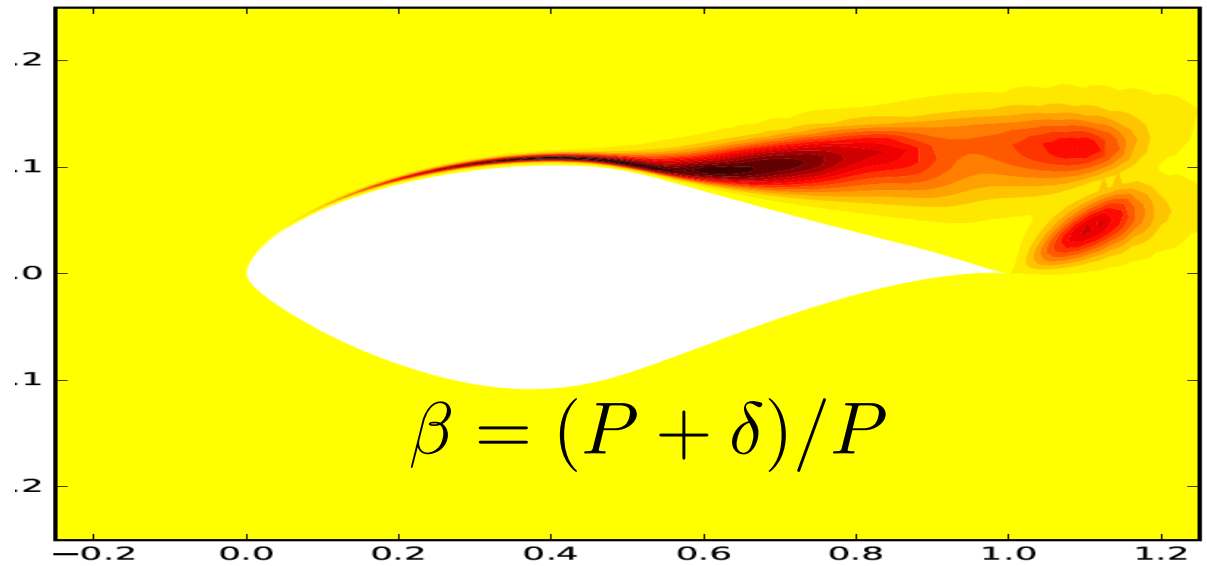
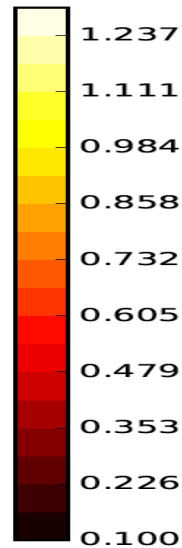
$$\frac{D\tilde{v}}{Dt} = P - D + T$$

## Introduce correction

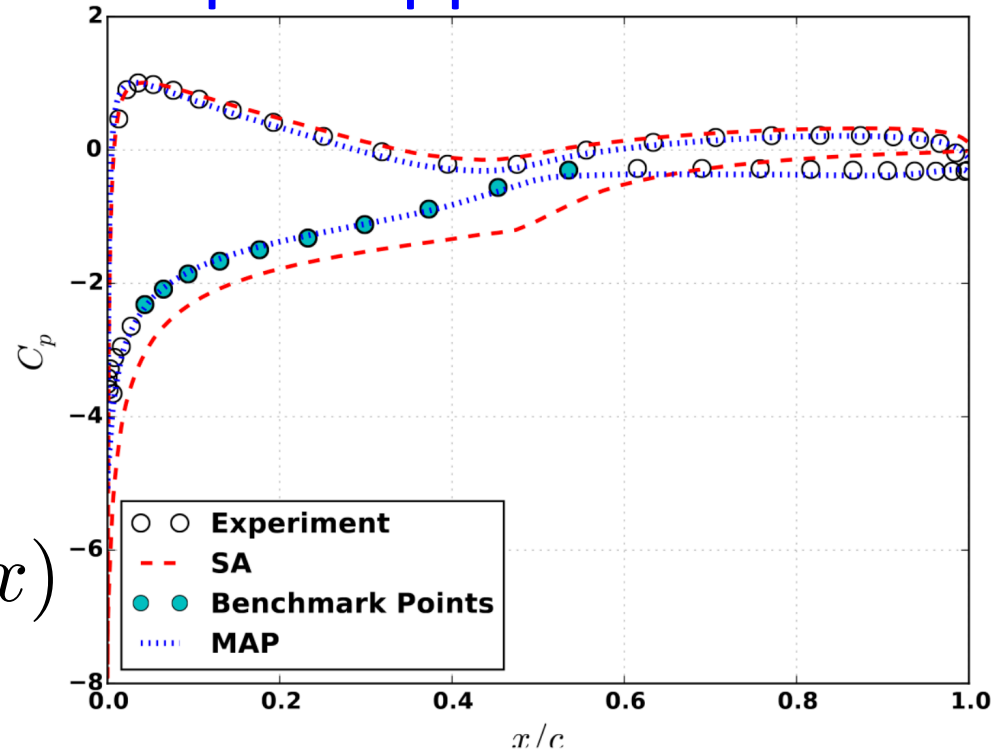
$$\frac{D\tilde{v}}{Dt} = P - D + T + \delta(x)$$

## Data used:

Wall  
pressure



## Example: Application to airfoil



# Bayesian FUNCTIONAL Inversion

$$\beta_{map} = \arg \min \frac{1}{2} \left[ (\mathbf{d} - h(\beta))^T \mathbf{C}_m^{-1} (\mathbf{d} - h(\beta)) + (\beta - \beta_{prior})^T \mathbf{C}_\beta^{-1} (\beta - \beta_{prior}) \right]$$

$\mathbf{d}$  – Data

$\beta$  - Unknown function

$h(\beta)$  – Model output

$\mathbf{C}_m$  - Observational covariance

$\mathbf{C}_\beta$  - Prior covariance

## Posterior

$$\mathbf{C}_{posterior} = \left[ \frac{d^2 \mathfrak{J}(\boldsymbol{\beta})}{d\boldsymbol{\beta} d\boldsymbol{\beta}} \right]^{-1} \bigg|_{\boldsymbol{\beta}_{MAP}}$$

$$H_{ij} = \frac{\partial^2 \mathfrak{J}}{\partial \beta_i \partial \beta_j} + \psi_m \frac{\partial^2 R_m}{\partial \beta_i \partial \beta_j} + \mu_{i,m} \frac{\partial R_m}{\partial \beta_j} + \nu_{i,m} \frac{\partial^2 \mathfrak{J}}{\partial u_n \partial \beta_j} + \nu_{i,n} \psi_m \frac{\partial^2 R_m}{\partial u_n \partial \beta_j}$$

where,

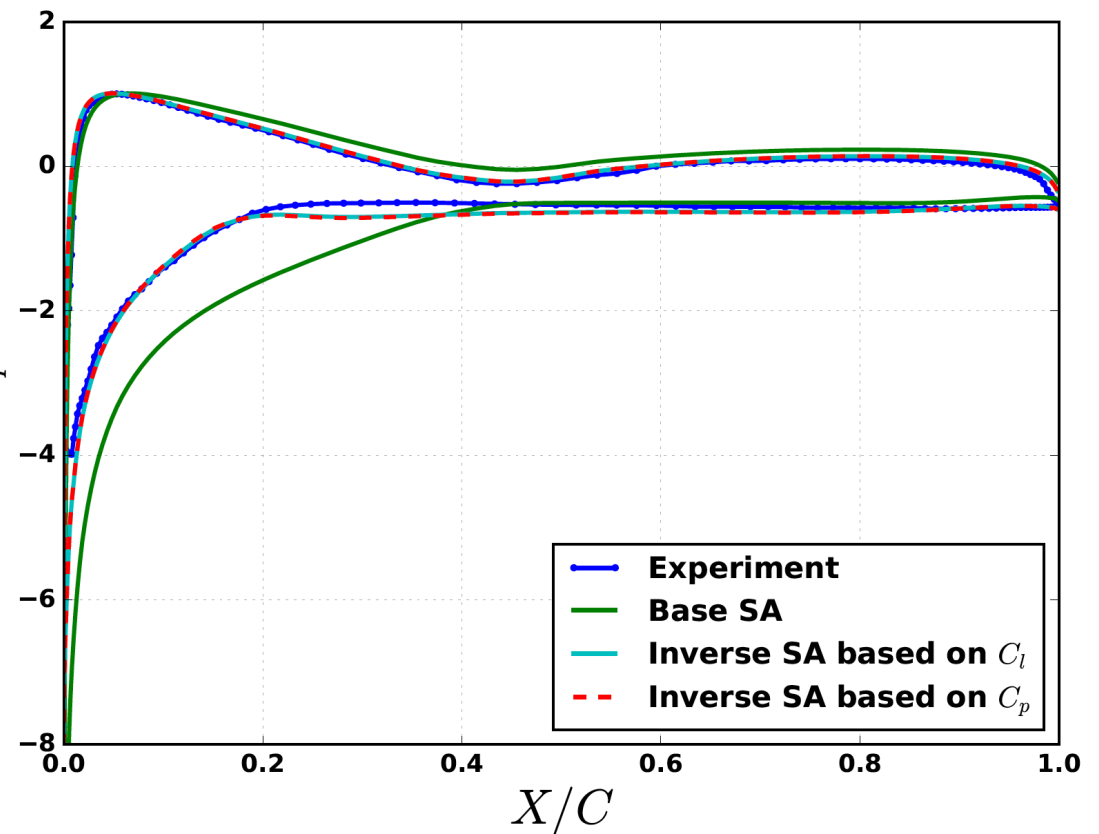
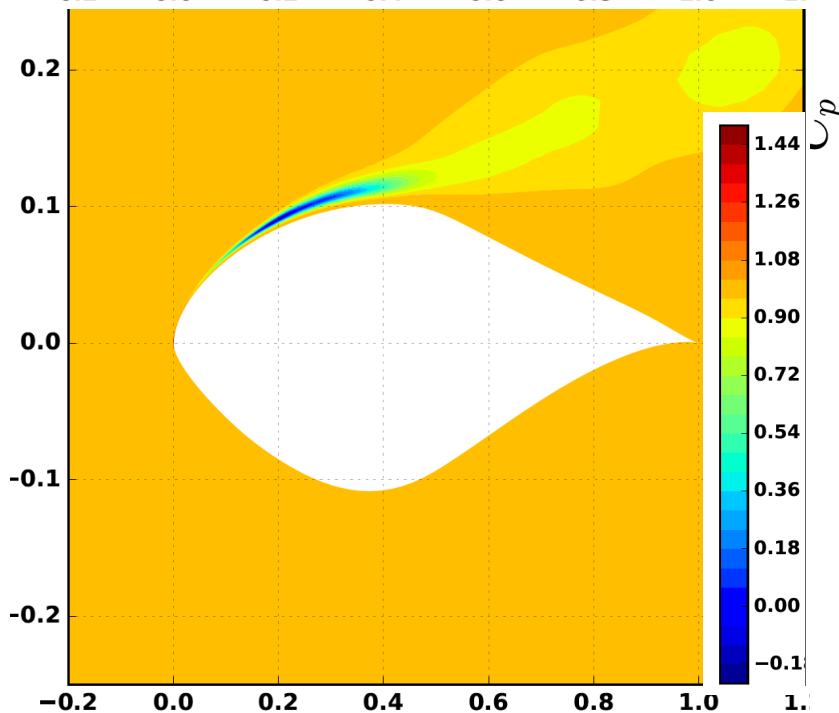
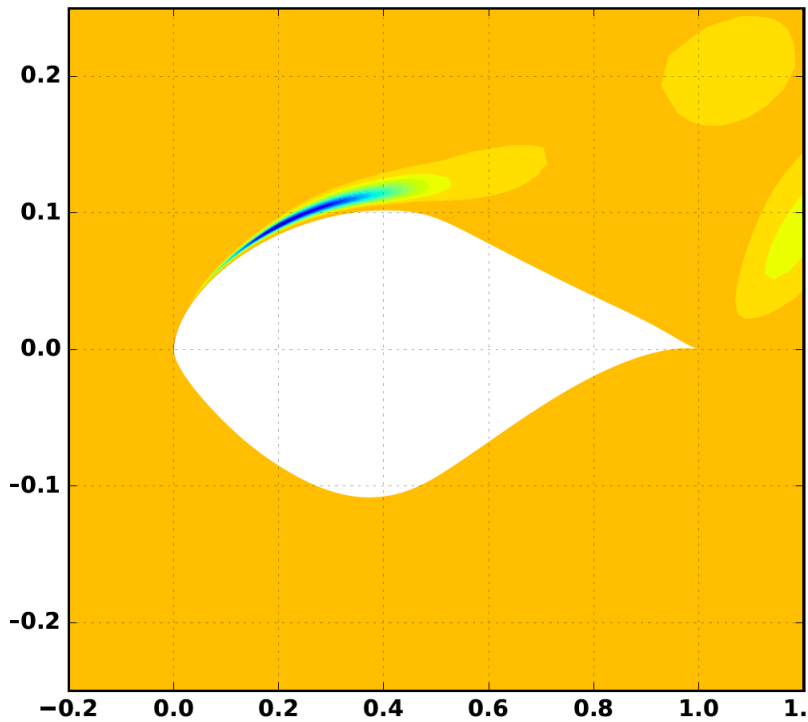
$$\nu_{i,n} \frac{\partial R_m}{\partial u_n} = - \frac{\partial R_m}{\partial \beta_i}$$

$$\mu_{i,m} \frac{\partial R_m}{\partial u_k} = - \frac{\partial^2 F}{\partial \beta_i \partial u_k} - \psi_m \frac{\partial^2 R_m}{\partial \beta_i \partial u_k} - \nu_{i,n} \frac{\partial^2 \mathfrak{J}}{\partial u_n \partial u_k} - \nu_{i,n} \psi_m \frac{\partial^2 R_m}{\partial u_n \partial u_k}$$

An approximate Hessian computation is additionally used for ill-posed problems

More complete PDFs with accelerated MCMC (with P. Constantine, Colorado Sc. Of Mines)

# Inversion based on Pressures vs Inversion based on LIFT!



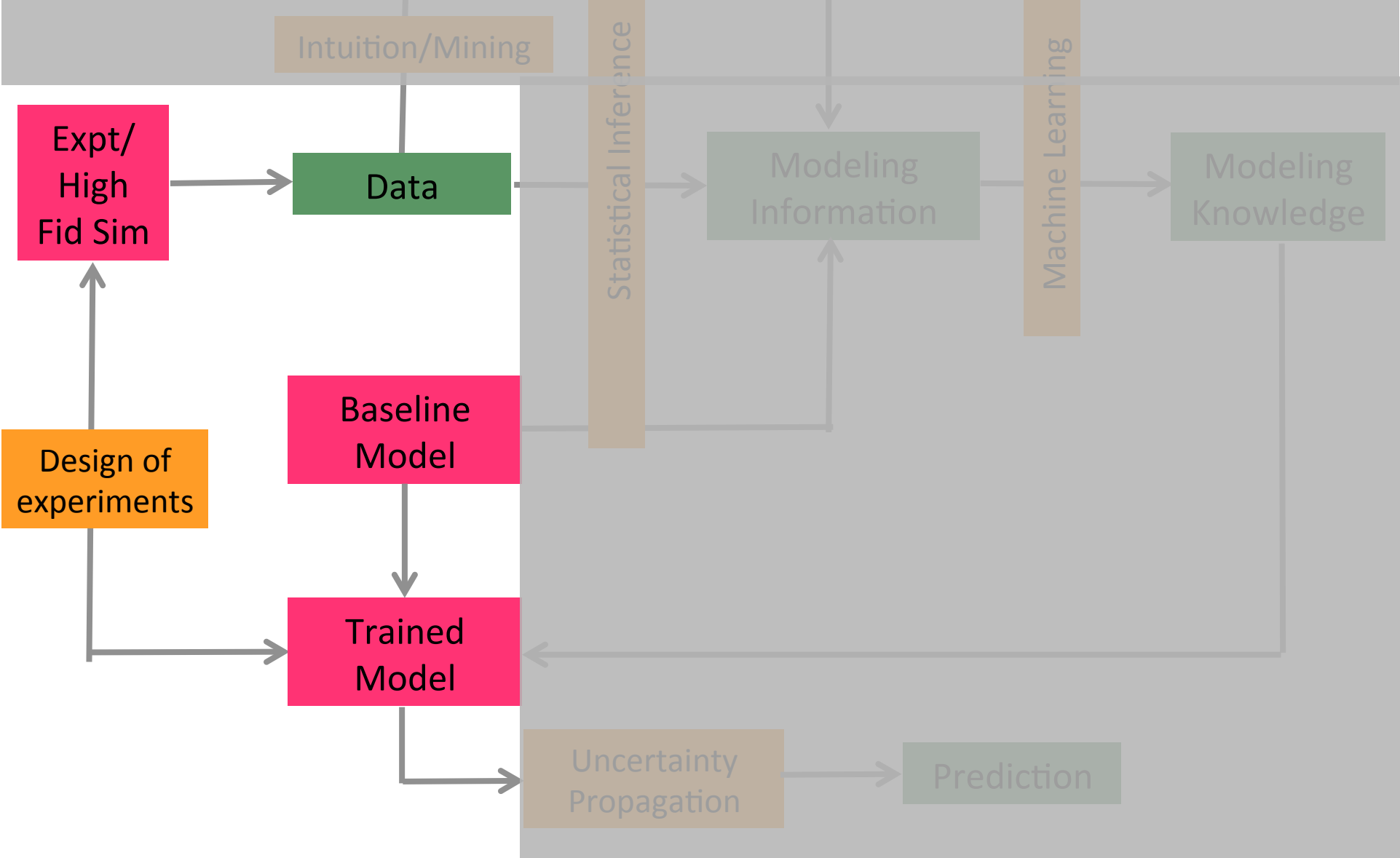
Ability to work on sparse amount of data is critical

## Other ways of introducing discrepancies

$$\frac{DR_{ij}}{Dt} = C_{ij} + P_{ij} + T_{ij} + \Pi_{ij} + D_{ij} + \beta(x)_{ij} \epsilon_{ij}$$

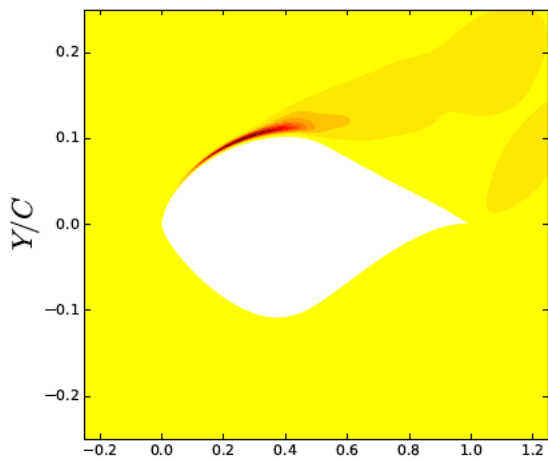
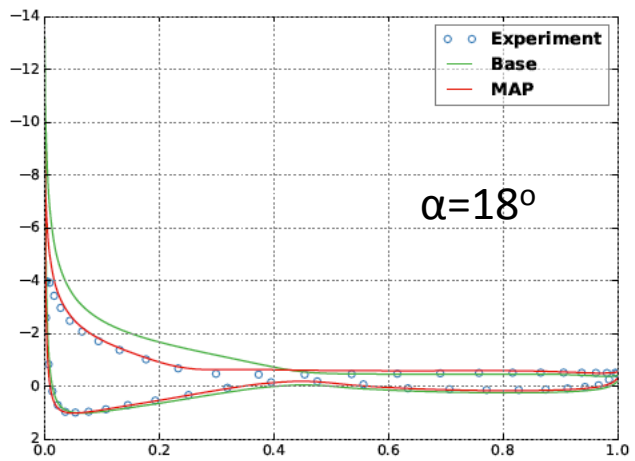
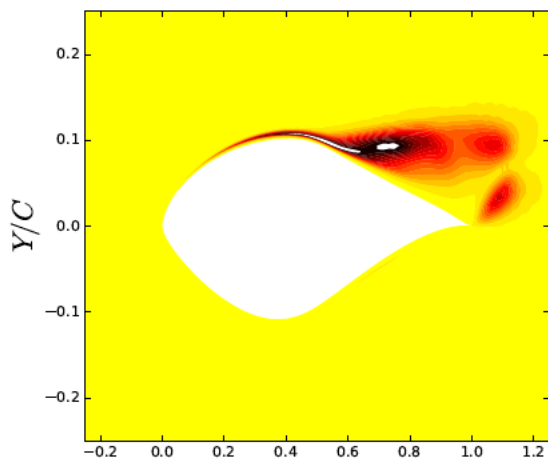
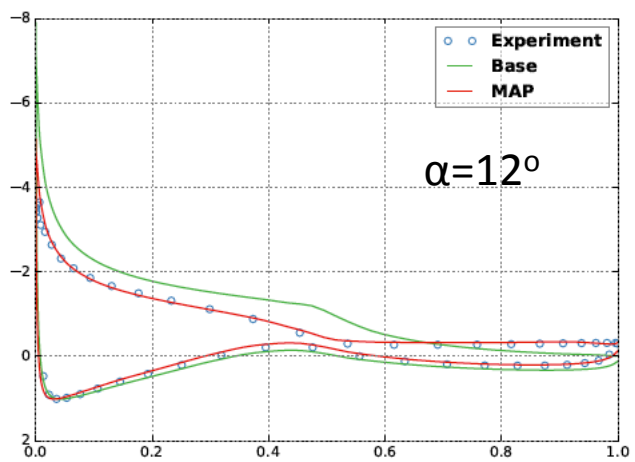
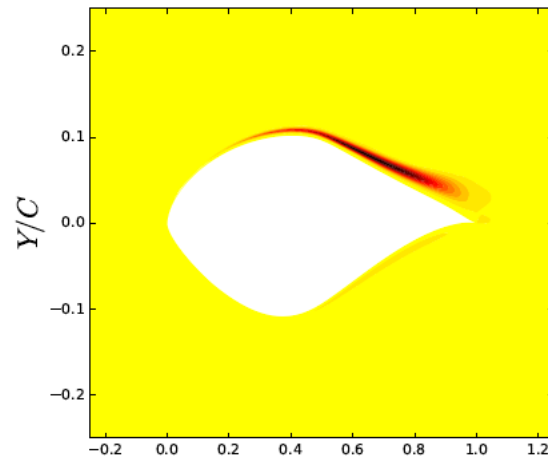
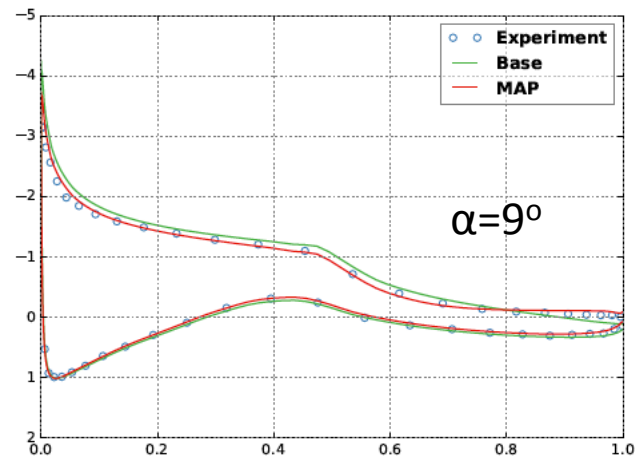
$$\frac{DR_{ij}}{Dt} = \beta(x)_{ij} a_o \omega (R_{ij,eq} - R_{ij})$$

$$\mathbf{R}_p = 2k \left[ \frac{\mathbf{I}}{3} + \mathbf{V} (\Lambda + \delta_\Lambda) \mathbf{V}^T \right]$$

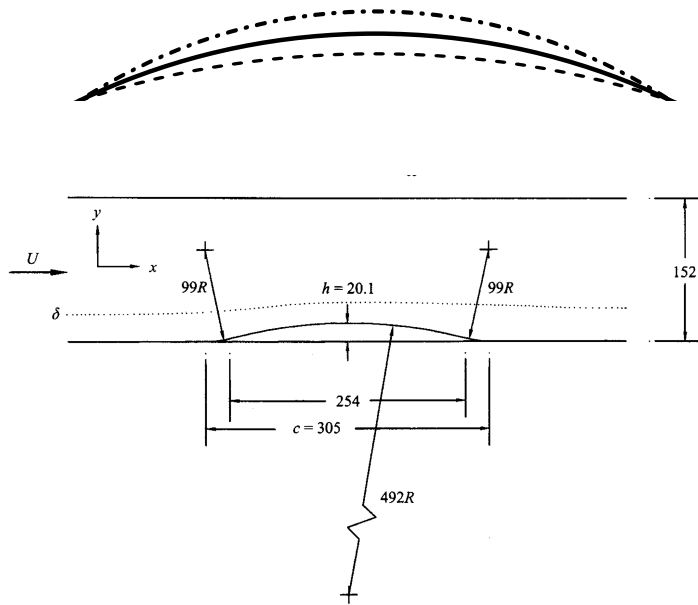




# Application to Airfoil flows

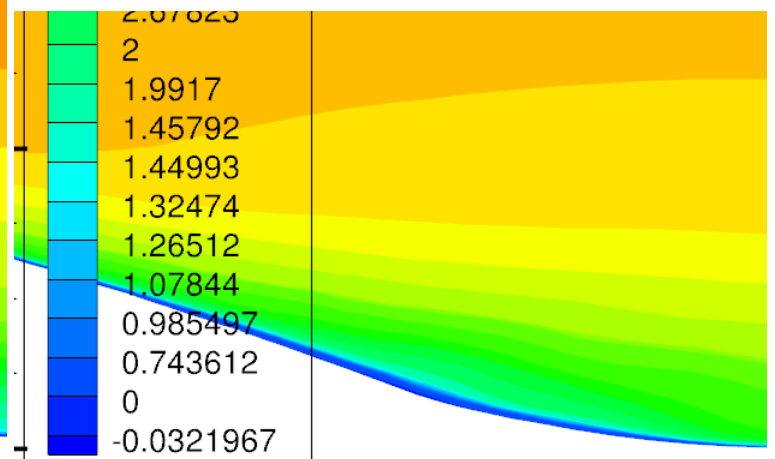
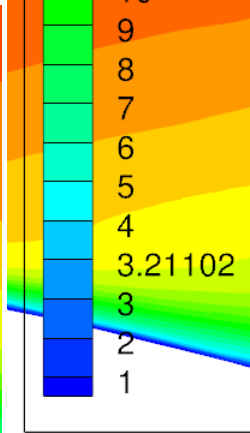
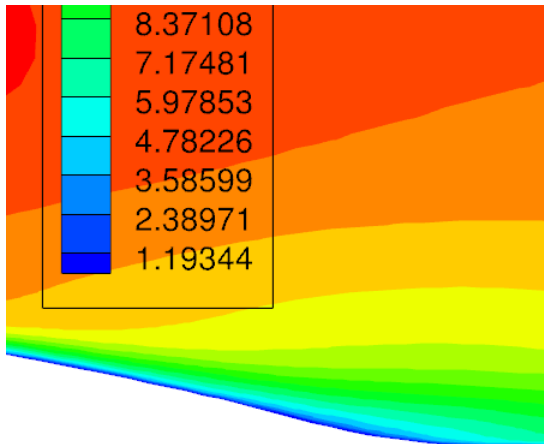


# Application to flows over bumps

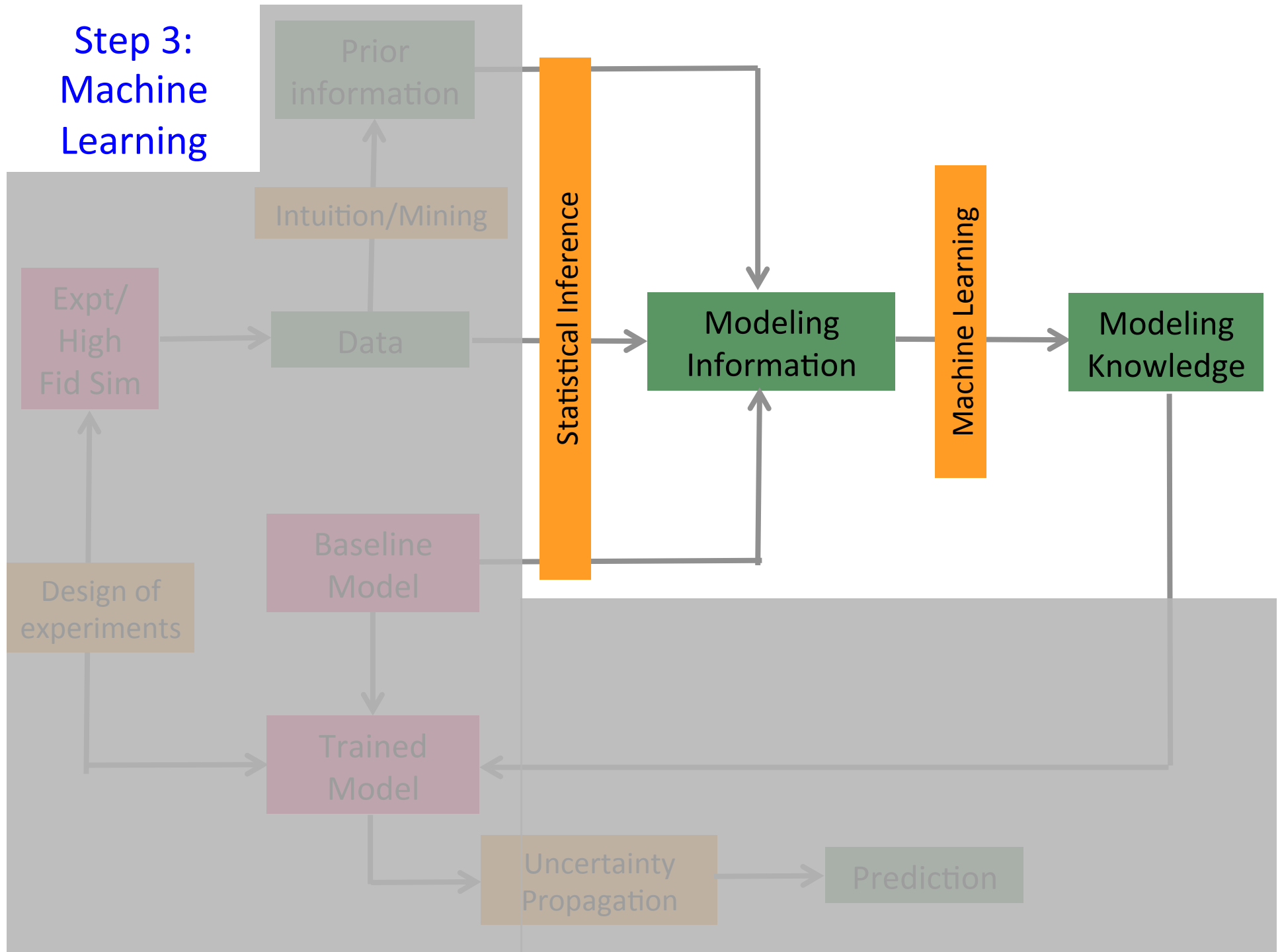


Large parameter sweep of different bump heights and Reynolds numbers

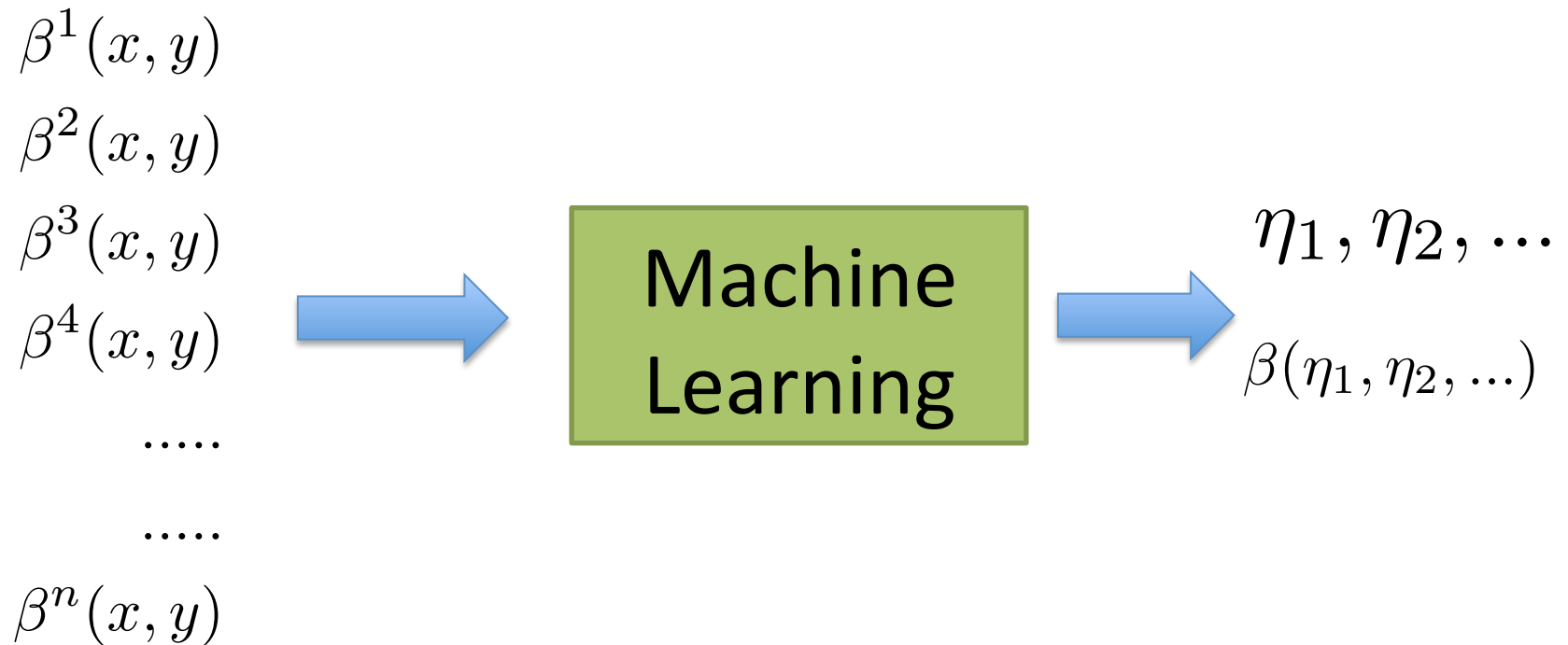
Targeting adverse pressure gradients and separation



### Step 3: Machine Learning



# How to transform information to knowledge?



# Selection of Features

Step 1: Look inside the baseline model

$$\chi = \hat{\mathbf{v}}/\mathbf{v} \quad \bar{\Omega} = \frac{d^2}{\hat{\mathbf{v}} + \mathbf{v}} \Omega$$

$$\bar{s}_p = \frac{d^2}{(\hat{\mathbf{v}} + \mathbf{v})^2} s_p = c_{b1}(1 - f_{t2}) \left( \frac{\chi}{\chi + 1} \right) \left( \bar{\Omega} + \frac{1}{\kappa^2} \frac{\chi}{\chi + 1} f_{t2} \right)$$

$$\bar{s}_d = \frac{d^2}{(\hat{\mathbf{v}} + \mathbf{v})^2} s_d = \left( \frac{\chi}{\chi + 1} \right)^2 c_{w1} f_w ,$$

Step 2: Look for relevant physics

$$S/\Omega, \Pi, s_p/s_d$$

Step 3: Feature-subset selection\*

Hill-climbing algorithm

Features locally non-dimensional

Kohavi, R. et al. "Wrappers for Feature Subset Selection," Artificial Intelligence, 1997

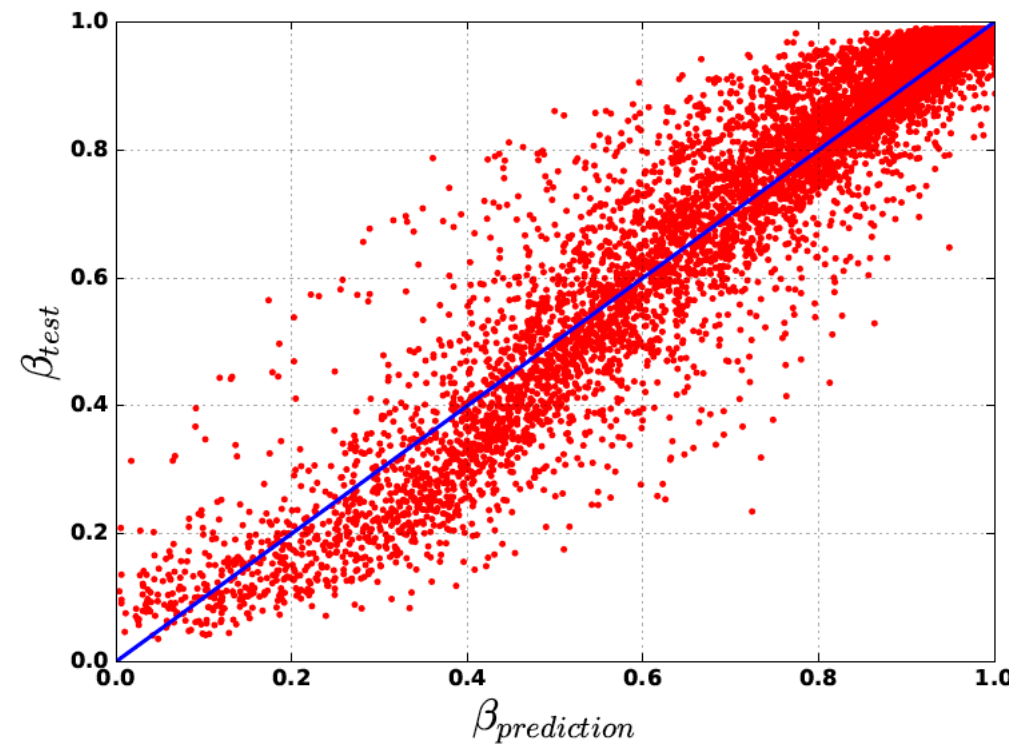
# Evaluation

*Neural Networks*

*GP regression*

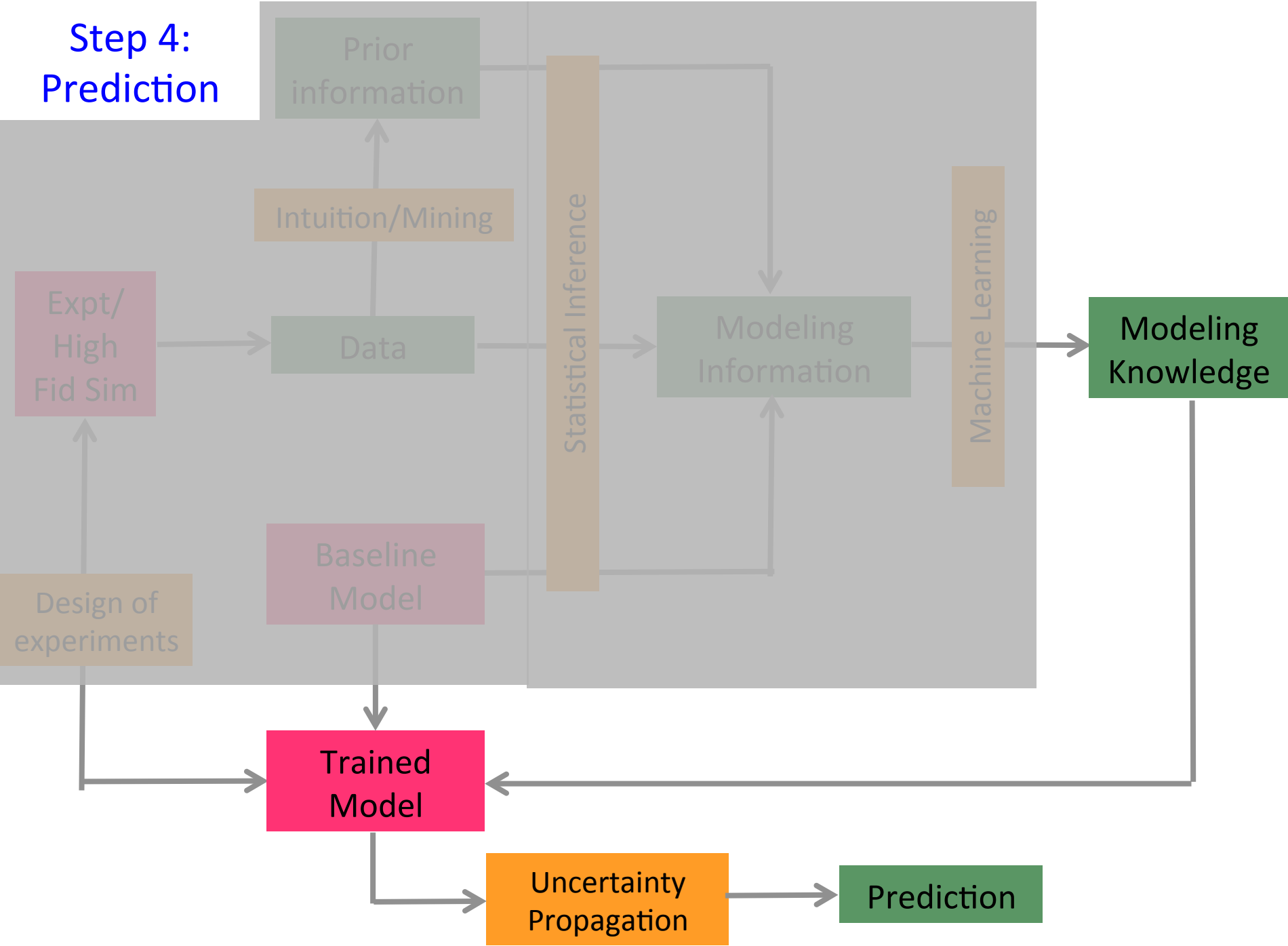
*Multiscale GP regression\**

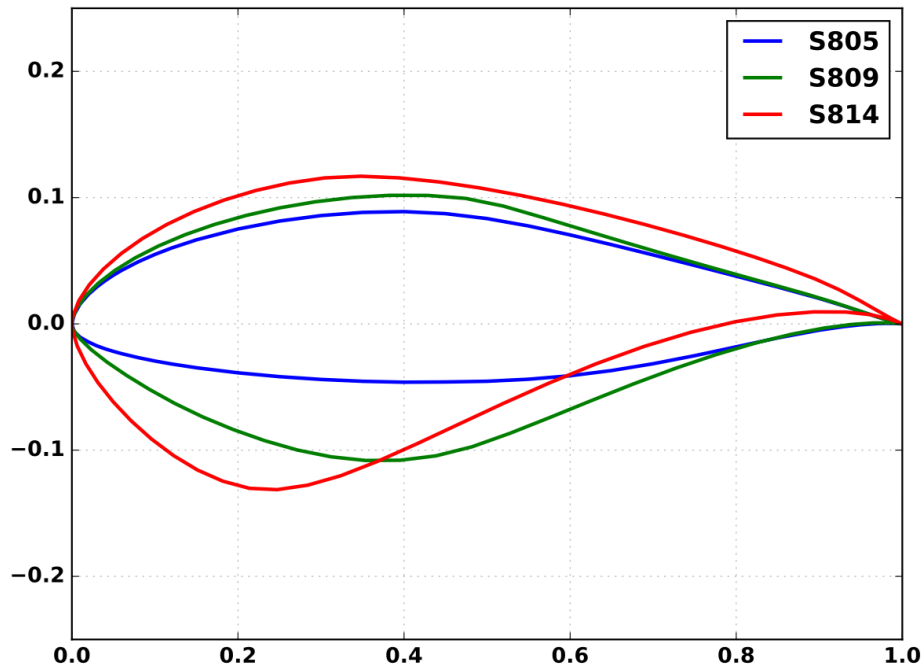
*Symbolic regression*



*\*Sparse Multiscale Gaussian Process Regression Using Hierarchical Clustering, Z. Zhang, K. Duraisamy, N. Gumerov, Applied Numerical Mathematics 2016 (Submitted)*

Step 4:  
Prediction





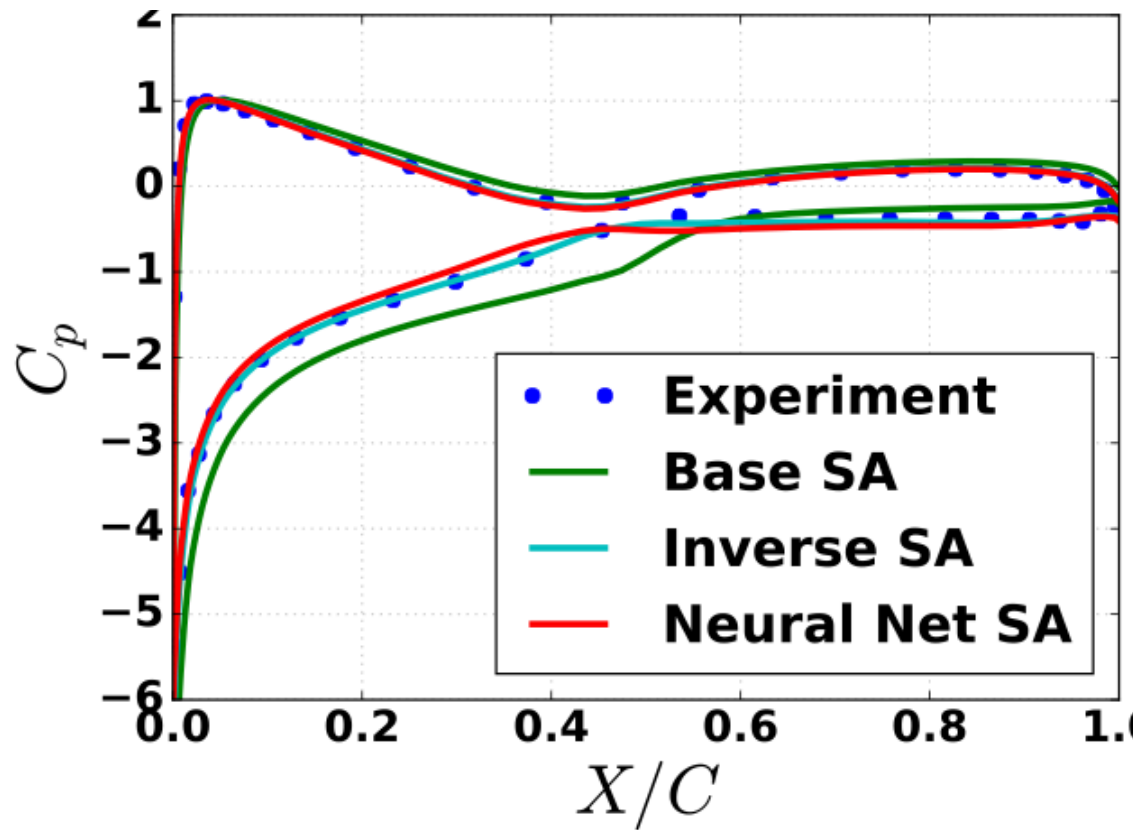
## Tests

Singh, A., Medida, S. & Duraisamy, K., [Data-augmented Predictive Modeling of Turbulent Separated Flows over Airfoils](#) Submitted, *AIAA Journal*, 2016.

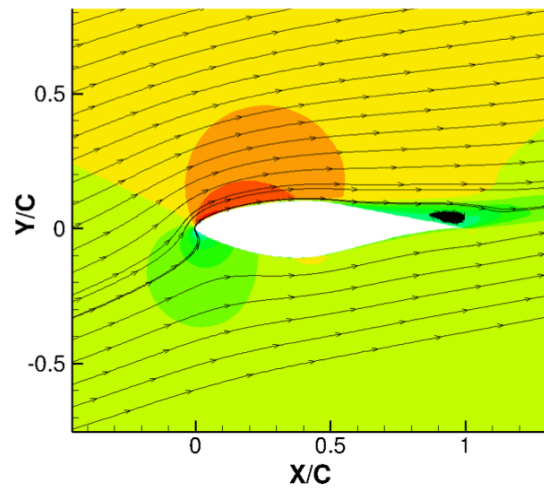
Model label	Training data
1	S805 at $Re = 1 \times 10^6$
2	S805 at $Re = 2 \times 10^6$
3	S809 at $Re = 1 \times 10^6$
4	S809 at $Re = 2 \times 10^6$
5	S805 at $Re = 1 \times 10^6, 2 \times 10^6$
6	S809 at $Re = 1 \times 10^6, 2 \times 10^6$
<b>P</b>	<b>S814 at <math>Re = 1 \times 10^6, 2 \times 10^6</math></b>
7	S805, S809, S814 at $Re = 1 \times 10^6, 2 \times 10^6$

Training set →

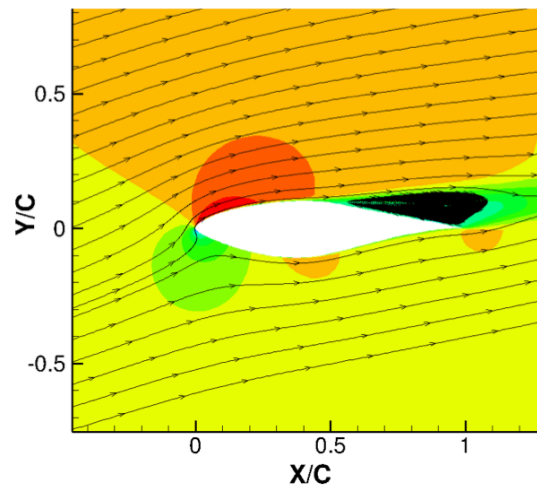




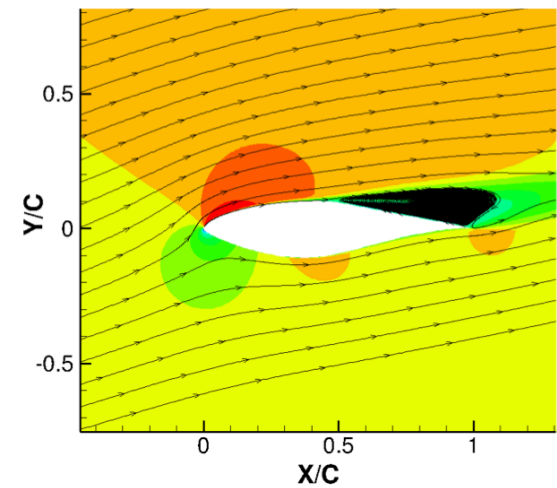
True prediction !



(a) Base SA

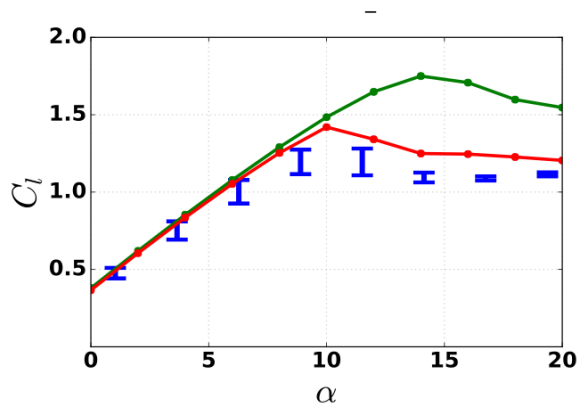


(b) Inverse SA

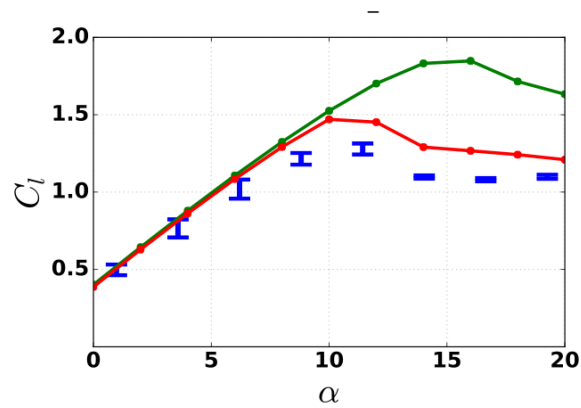


(c) NN-augmented SA (prediction)

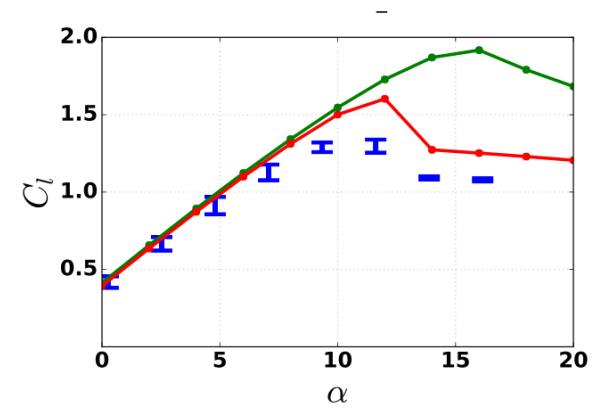
# Prediction – S814



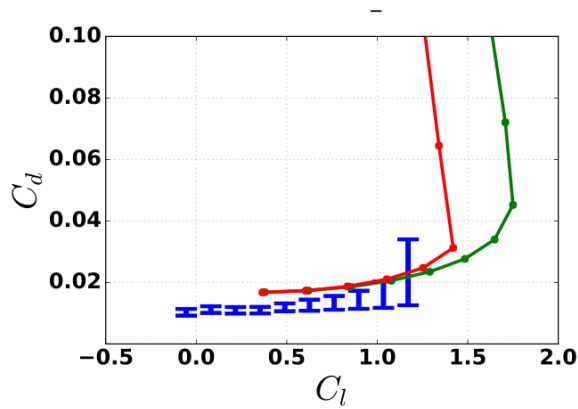
(a)  $Re = 1 \times 10^6$



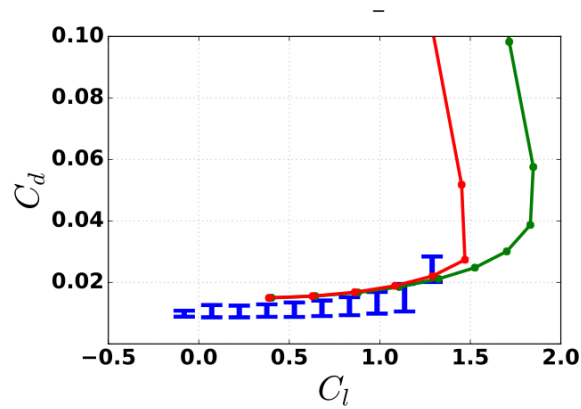
(b)  $Re = 2 \times 10^6$



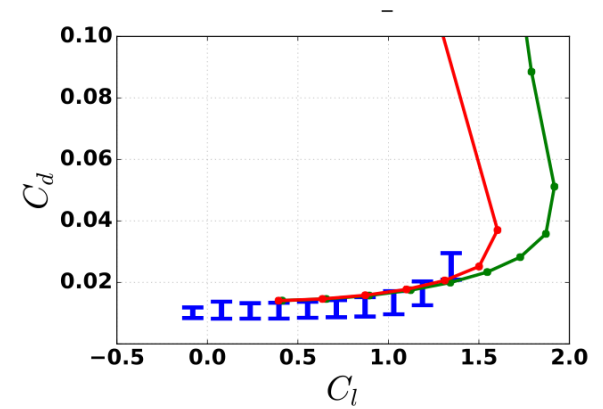
(c)  $Re = 3 \times 10^6$



(d)  $Re = 1 \times 10^6$



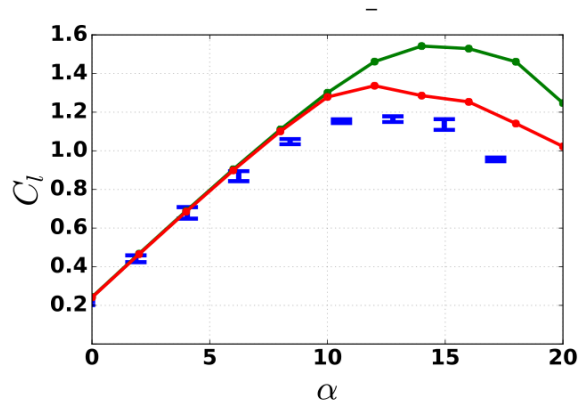
(e)  $Re = 2 \times 10^6$



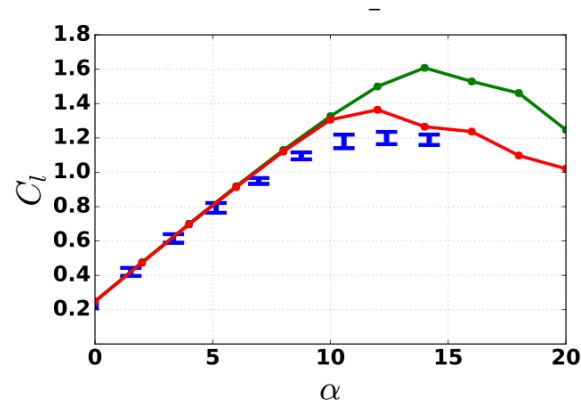
(f)  $Re = 3 \times 10^6$

Collaboration with Altair, Inc.

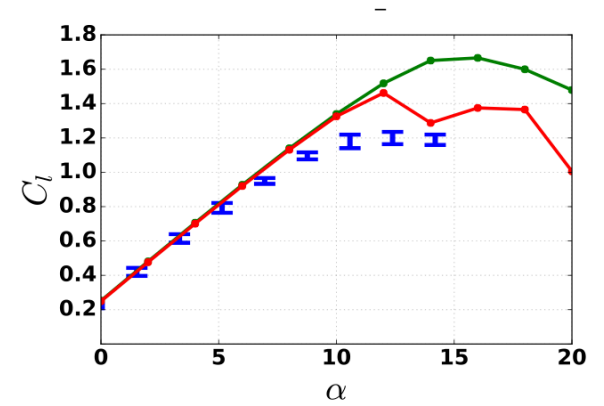
# Prediction – S805



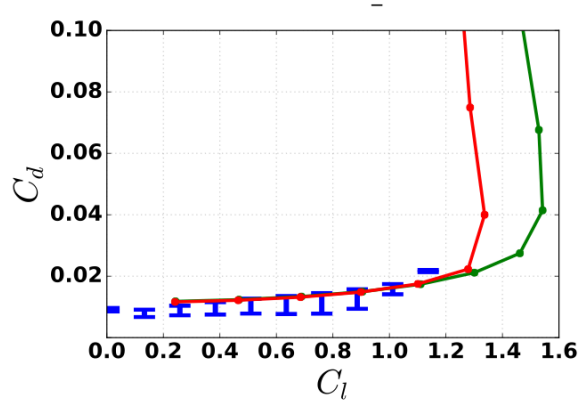
(a)  $Re = 1 \times 10^6$



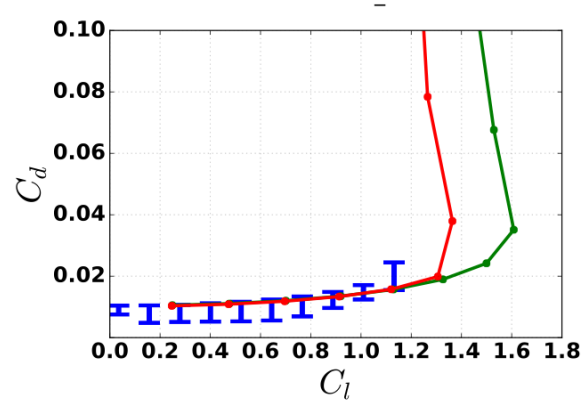
(b)  $Re = 2 \times 10^6$



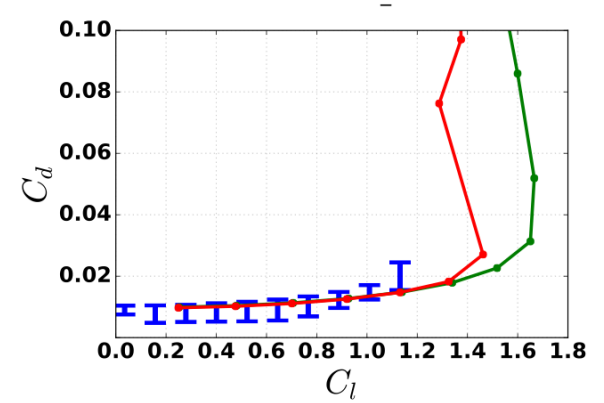
(c)  $Re = 3 \times 10^6$



(d)  $Re = 1 \times 10^6$



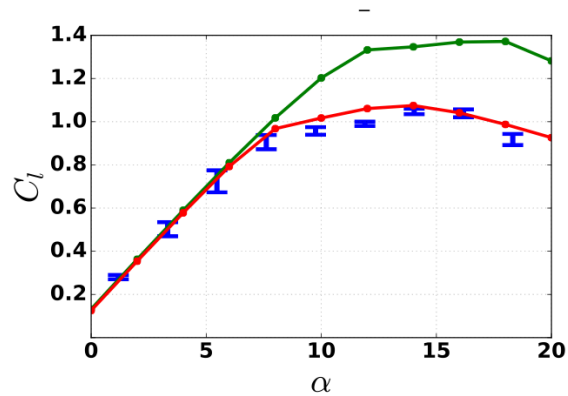
(e)  $Re = 2 \times 10^6$



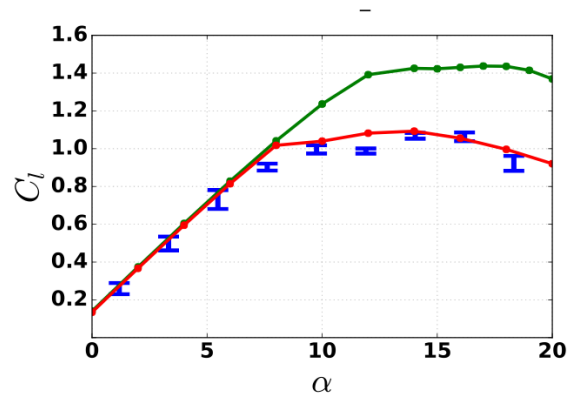
(f)  $Re = 3 \times 10^6$

Collaboration with Altair, Inc.

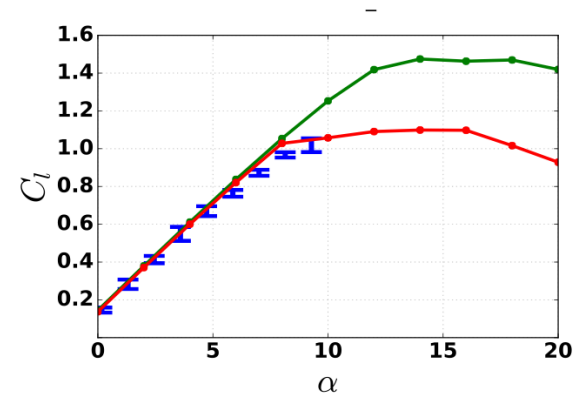
# Prediction – S809



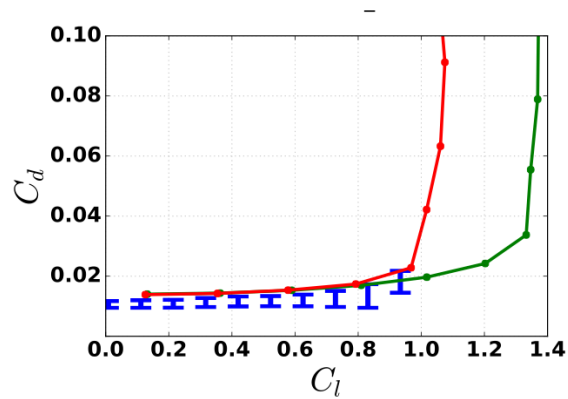
(a)  $Re = 1 \times 10^6$



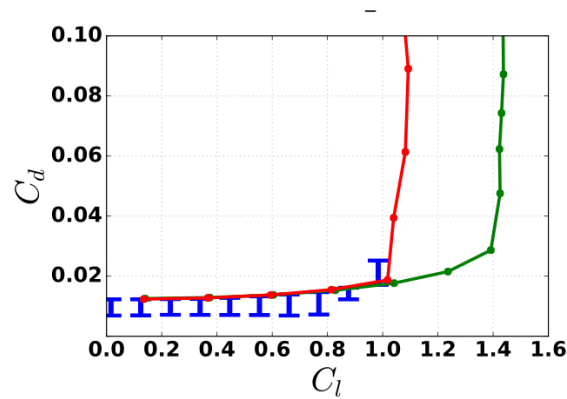
(b)  $Re = 2 \times 10^6$



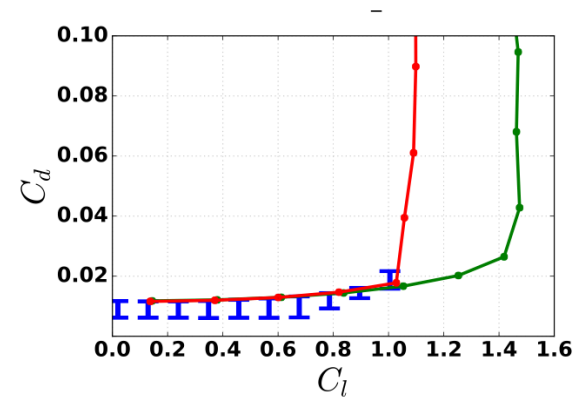
(c)  $Re = 3 \times 10^6$



(d)  $Re = 1 \times 10^6$



(e)  $Re = 2 \times 10^6$

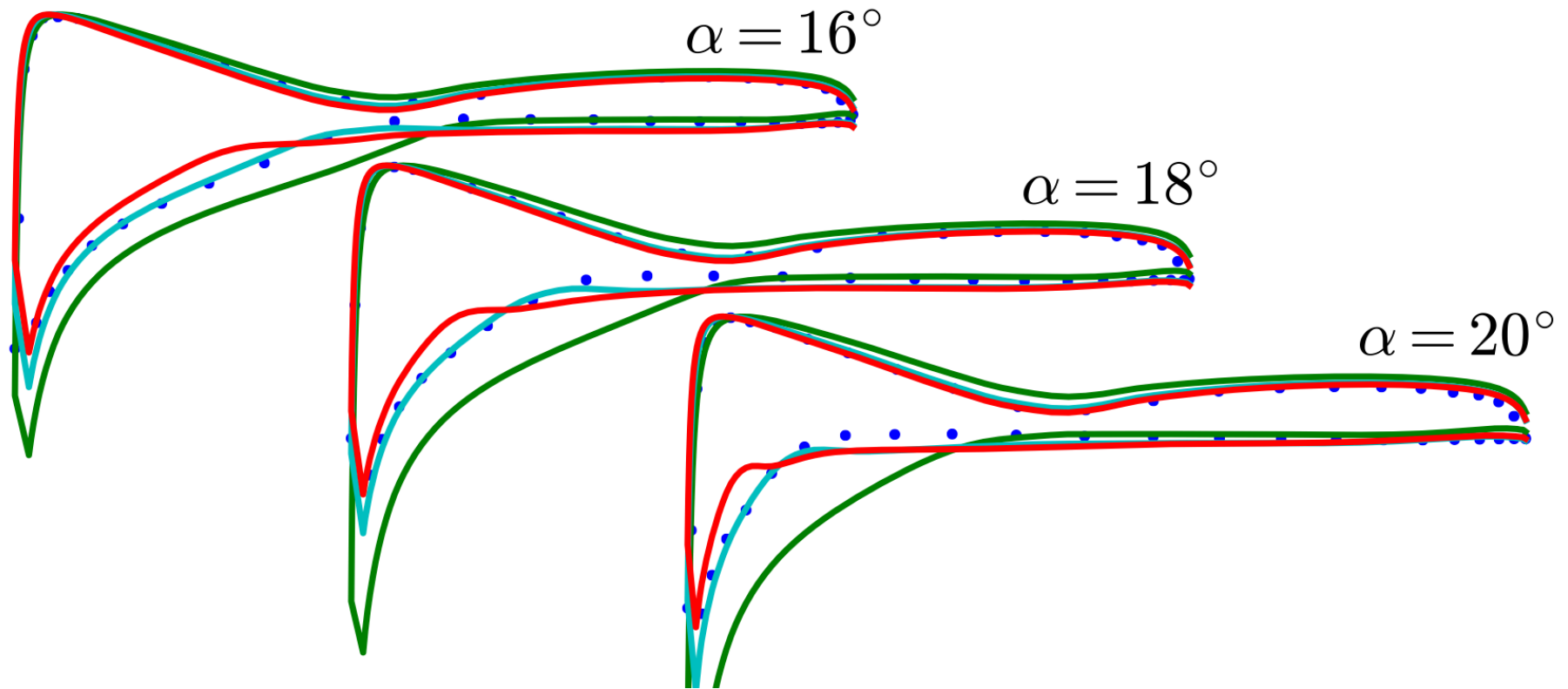


(f)  $Re = 3 \times 10^6$

Collaboration with Altair, Inc.

# True prediction !

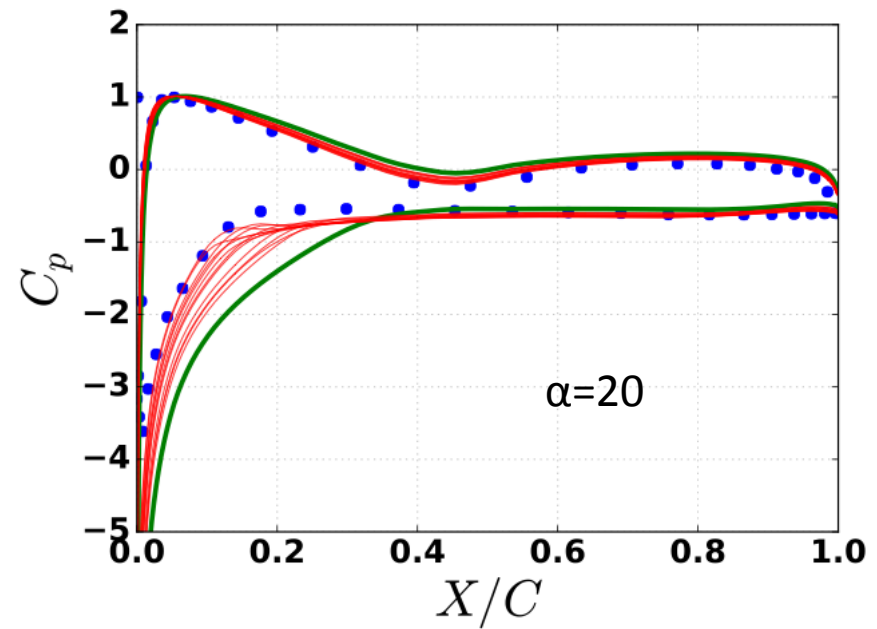
S 809, Re=2 Million



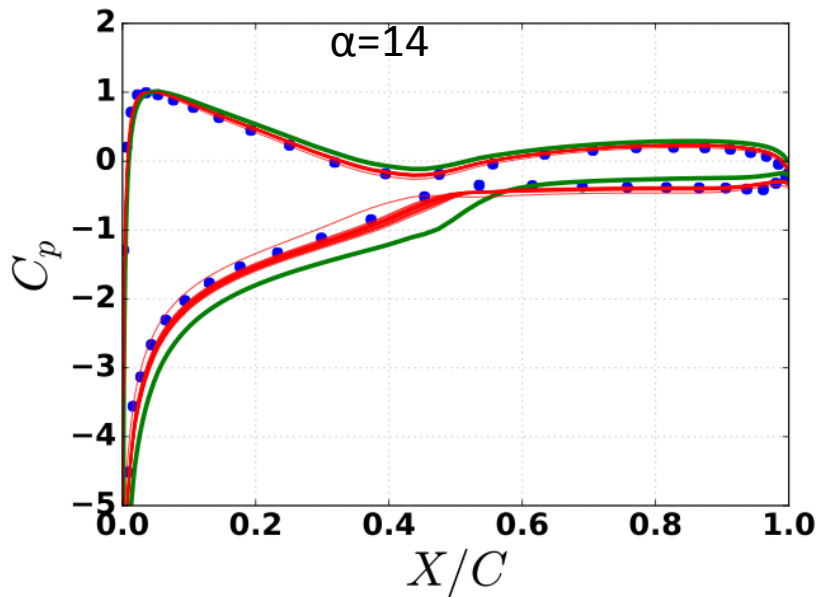
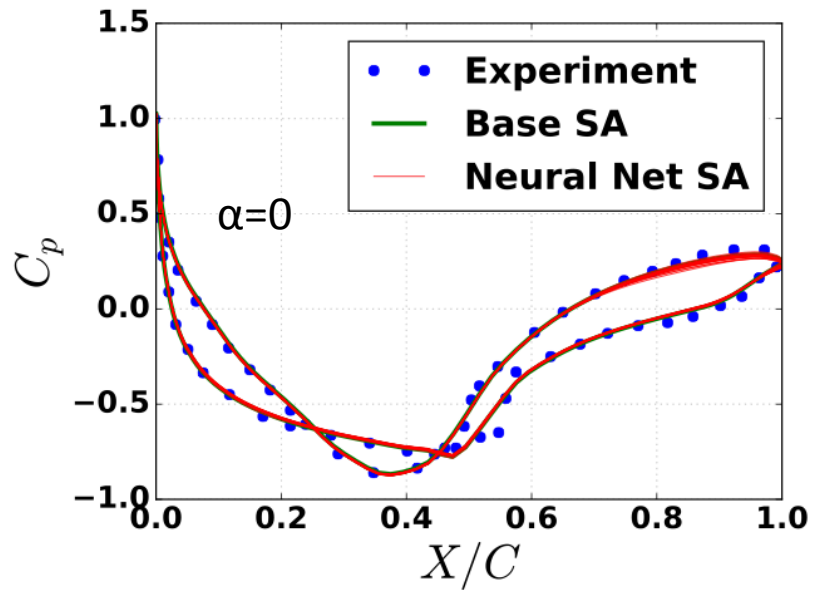
Inference used only CL data, NN-augmented model provides considerable predictive improvements of  $C_p$

## Variability

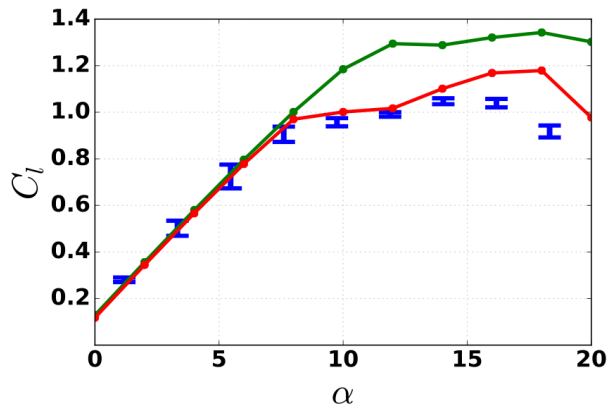
S 809, Re=2 Million



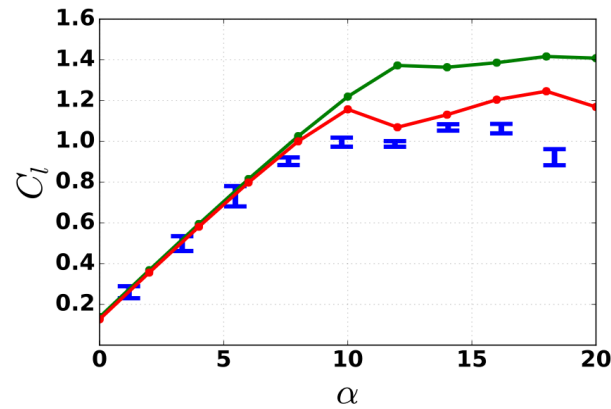
Training from different  
sets



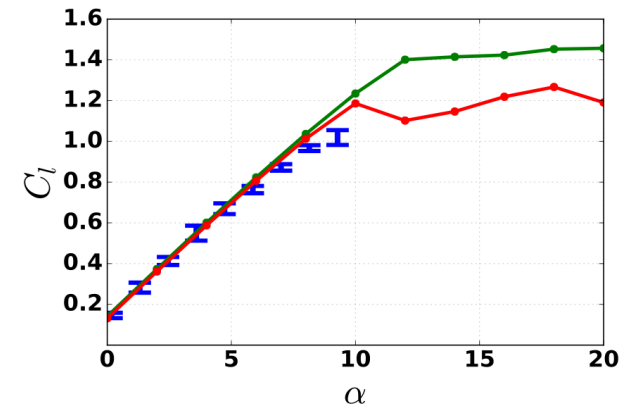
# Portability : Implementation in AcuSolve



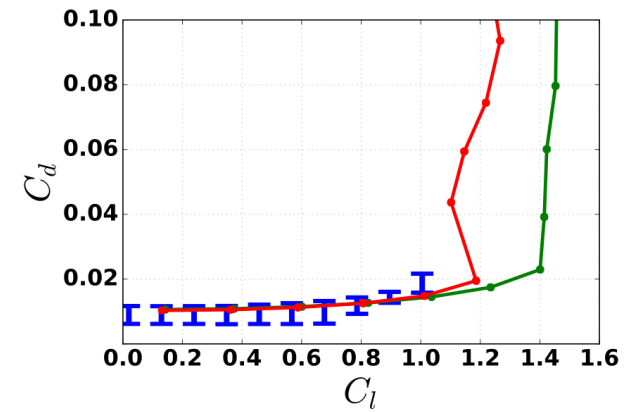
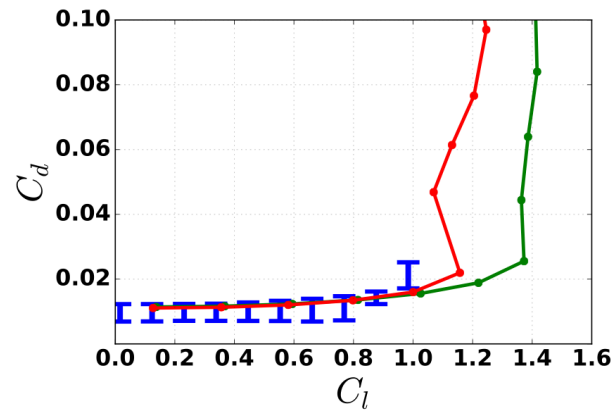
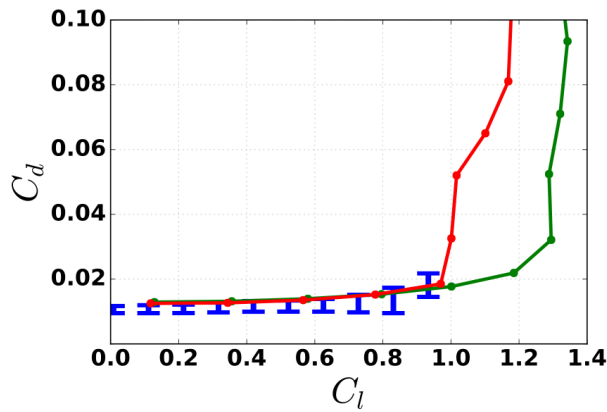
(a)  $Re = 1 \times 10^6$



(b)  $Re = 2 \times 10^6$

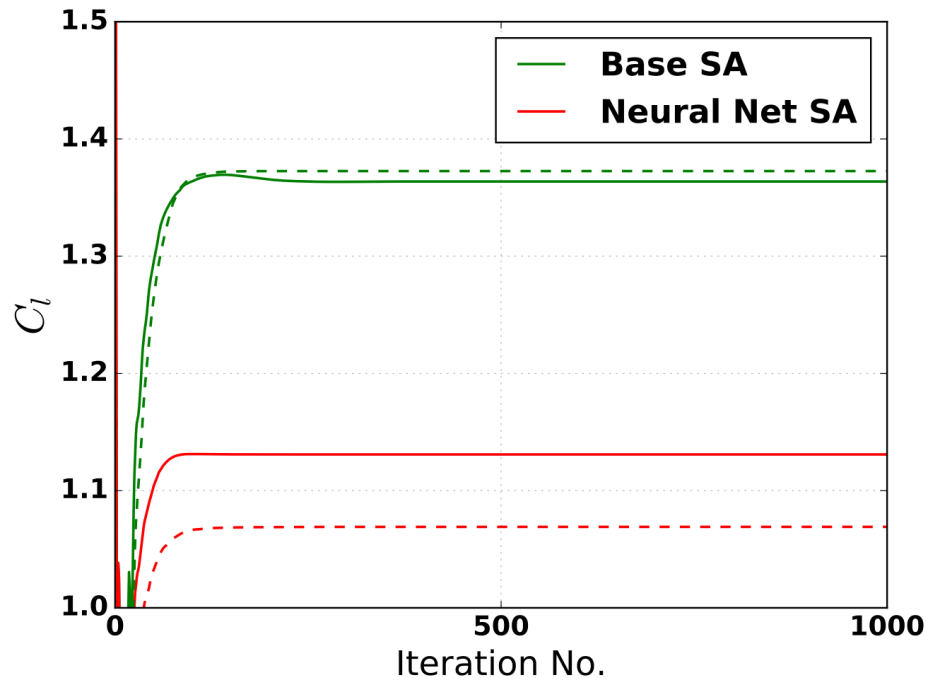


(c)  $Re = 3 \times 10^6$

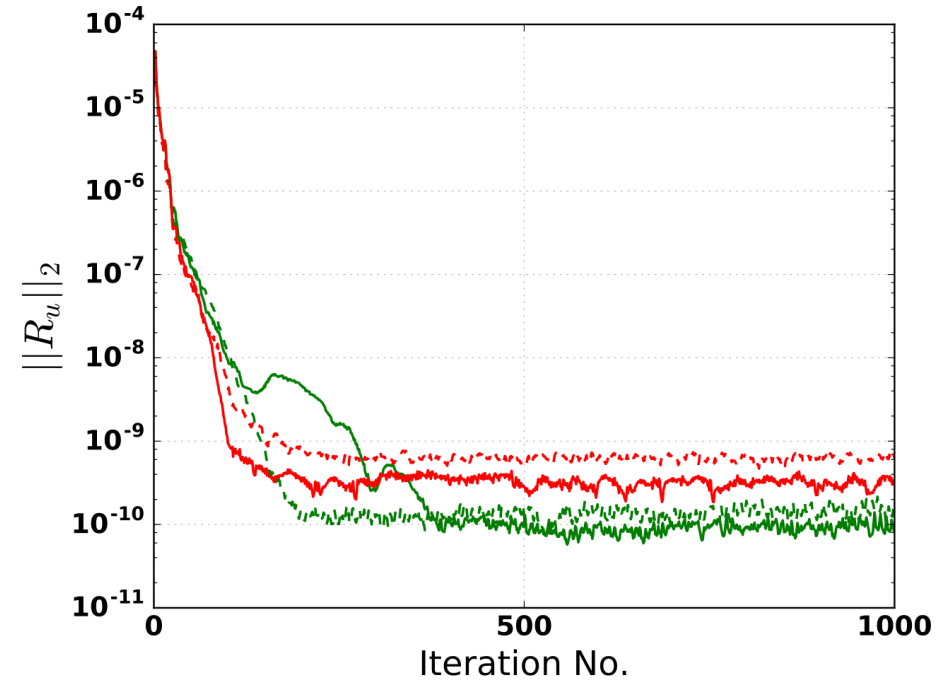


S809 Airfoil : Predictive results in Commercial CFD solver

## Robustness: Implementation in AcuSolve



(a) Lift coefficient



(b) L2 norm of solver residual

S809 Airfoil : Predictive results in Commercial CFD solver



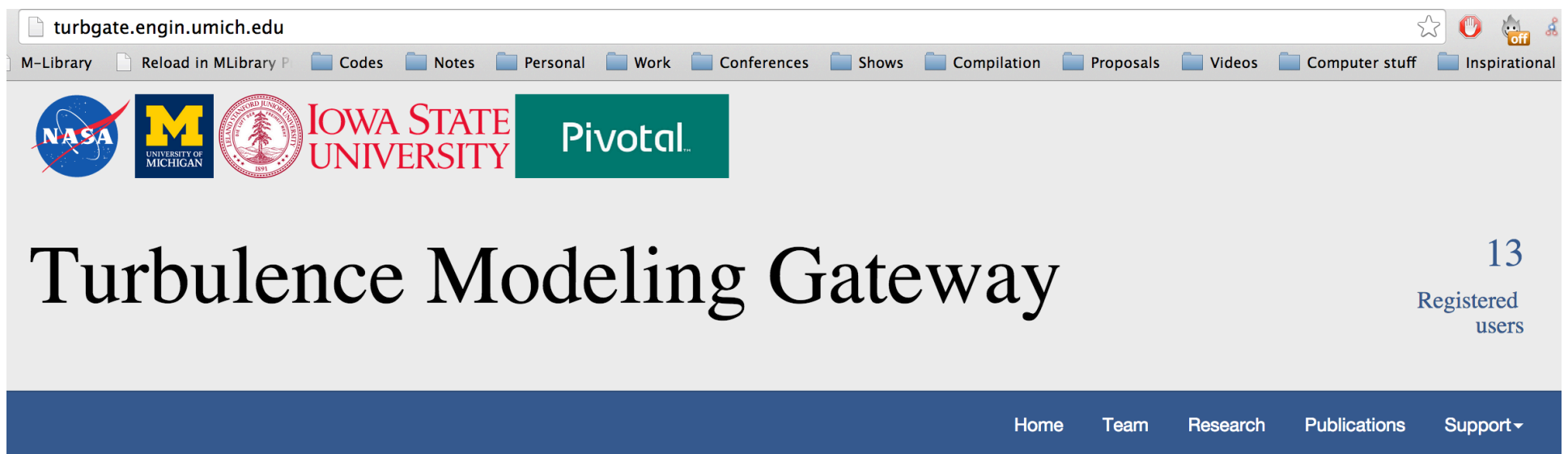
## Summary of predictions

- NN-augmented S-A model shows **significant improvement** over the baseline model in predicting CL , CD and stall onset angles.
- Predictions confirmed to be excellent **in shapes and flow conditions** that were not part of the training set.
- No **deterioration** of accuracy noticed in situations in which original model was accurate
- **Inference used only CL** but NN-augmented model provides considerable **predictive improvements of Cp**
- **Ensemble of predictions** used to assess the sensitivity of the model outputs to training
  - ➔ Currently introducing uncertainties via GPs
- **Solver convergence was assessed** and typically the cost overhead for the NN augmentations ~ 10-30%
- **Portability** addressed by developing framework in one solver and injecting in a commercial solver (Acusolve)

# Vision for the future

A continuously augmented curated database / website of inferred corrections that are input to the machine learning process

Users upload/download/process data, generate maps.



Welcome to the Turbulence Modeling Gateway Server. The goal of our project is to develop the science behind data driven turbulence modeling and demonstrate the utility of large-scale data-driven techniques in turbulence modeling. Our work involves the development of domain-specific learning techniques suited for the representation of turbulence and its modeling, the establishment of a trusted ensemble of data for the creation and validation of new models, and the deployment of these models in complex aerospace problems. We are funded by the LEARN (Leading Edge Aeronautics Research for NASA) program, through the NASA Aeronautics Research Institute (NARI).

This is a collaborative effort between the University of Michigan, Stanford University, Iowa State and Pivotal Inc. We also consult with Boeing Commercial Airplanes and interact with NASA Langley Research Center.

## Email

## Password

## Growing community for data-driven turbulence modeling

2013- onwards: Duraisamy et al

2015- onwards: Ling et al. (apriori → modeling)

2015- onwards: Weatheritt et al (apriori)

2016- onwards: Xiao et al. (“open box UQ” → data-driven)

2016- onwards: Dwight et al. (parametric → non-parametric)

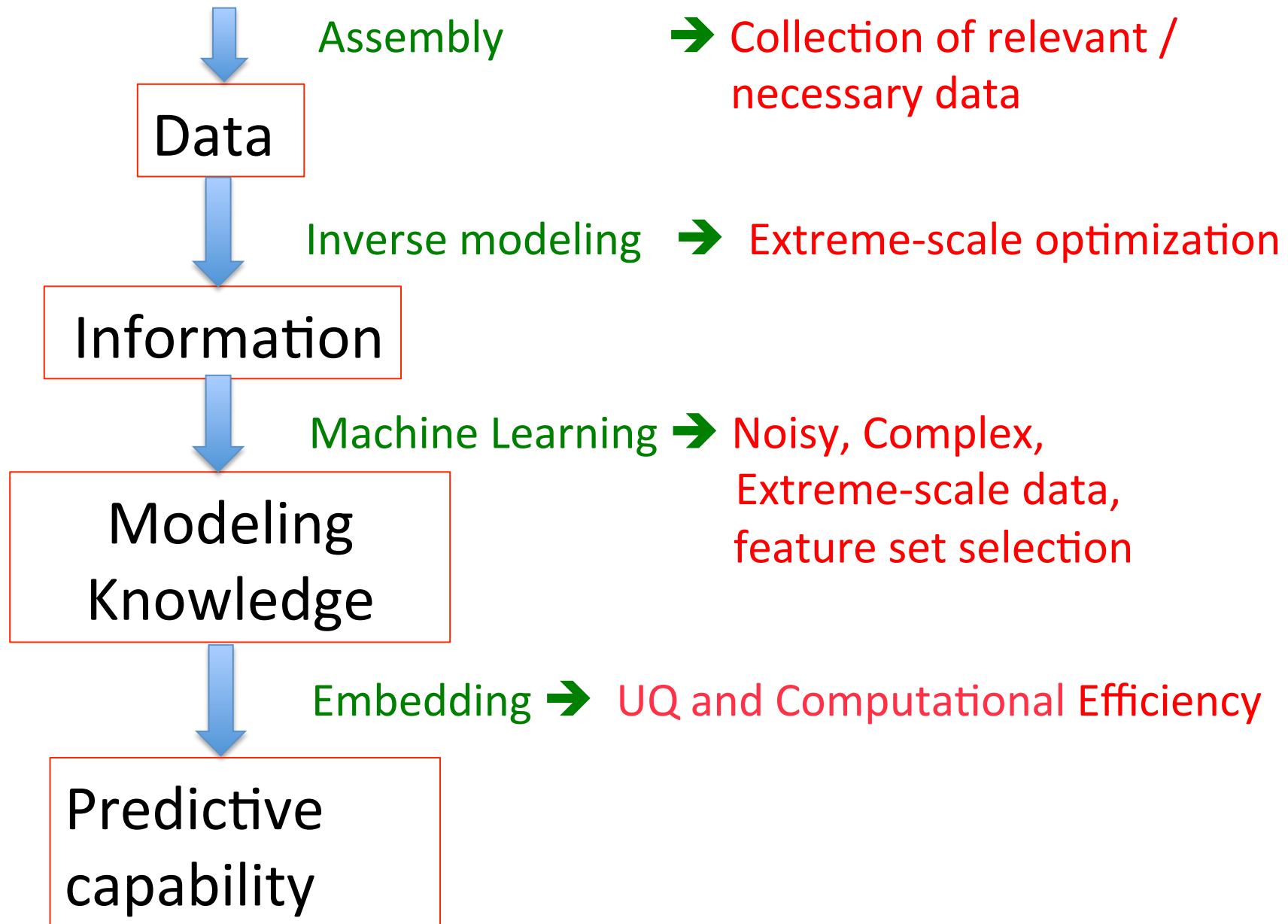
2016- onwards: Wang et al. (UQ → closure improvement)

Others: Moser, Edeling, Cinella, etc.

Companies: Altair, Inc. ; CD-Adapco; GE; Boeing

**Thanks to NASA for getting it started**

# Challenges



# Implications & Impact

**Data:** Very limited **experimental** data (more DNS/LES data when available)

**Modeling insight :** Modeler can understand what the model lacks to match data. This is done within the context of the model

**Improved models :** Can learn the missing components of model and generate improved models

**Model credibility:** Can validate/invalidate model structures

**Uncertainty quantification:** Can obtain modeling error bounds →  
Demonstrated in 1D problem

We are applying this framework within the context of turbulent flow, materials modeling and astrophysics.

# References

Singh, A., Medida, S. & Duraisamy, K., [Data-augmented Predictive Modeling of Turbulent Separated Flows over Airfoils](#) [Submitted, AIAA Journal](#), 2016.

Zhang, Z. & Duraisamy, K. & Gumerov, N. [Sparse Multiscale Gaussian Process Regression using Hierarchical Clustering](#), [Submitted, Applied Numerical Mathematics](#), 2016

Singh, A. & Duraisamy, K. [Using Field Inversion to Quantify Functional Errors in Turbulence Closures](#), [Physics of Fluids](#), 2016

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Zhang, Z. & Duraisamy, K., [Machine Learning Methods for Data-Driven Turbulence Modeling](#), [AIAA Aviation](#), 2015.

Tracey, B. & Duraisamy, K., & Alonso, J. [A Machine Learning Strategy to Assist Turbulence Model Development](#), [AIAA SciTech](#), 2015.

Duraisamy, K.; Zhang, Z. & Singh, A, [New Approaches in Turbulence and Transition Modeling Using Data-driven Techniques](#), [AIAA SciTech](#), 2015

Duraisamy, Karthik & Durbin , P.A., [Transition modeling using data driven approaches](#), [Proc. of the CTR Summer Program](#), 2014.

# Looking for post-docs and grad students

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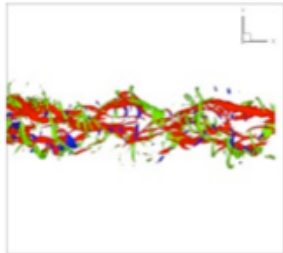
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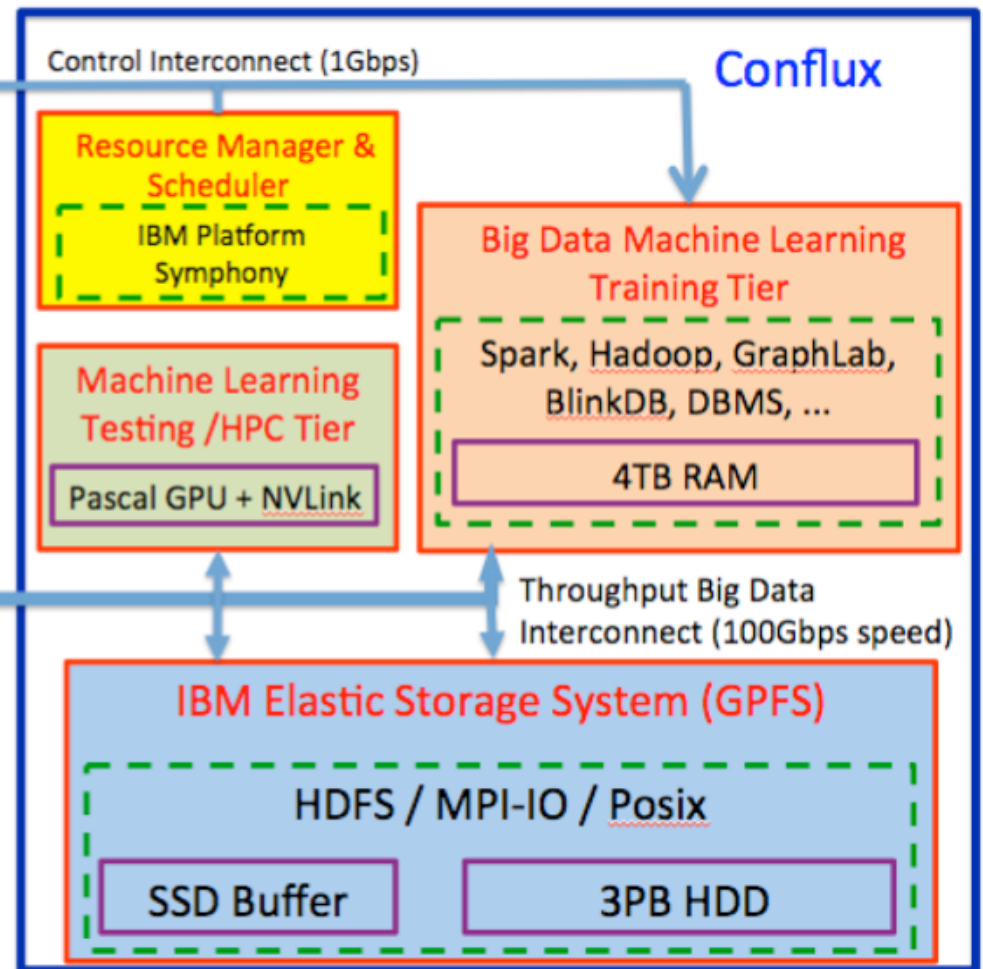
## U of Michigan Project Combines Modeling and Machine Learning

Tiffany Trader



Although we've yet to settle on a term for it, the convergence of HPC and a new generation of big data technologies is set to transform science. The compute-plus-data mantra reaches all the way to the White House with President Obama's National Strategic Computing Initiative calling for useful exascale computing and sophisticated data capabilities that serve the nation's overarching goals around security, innovation and competitiveness.

A champion of this paradigm, the National Science Foundation has been directing its resources toward providing the infrastructure and tools necessary to advance data-driven science at multiple scales.



*Center for Data-driven Computational Physics, University of Michigan*

# Backups



# Engineering simulation of turbulent flows

What we can compute

Ensemble/time averaged quantities

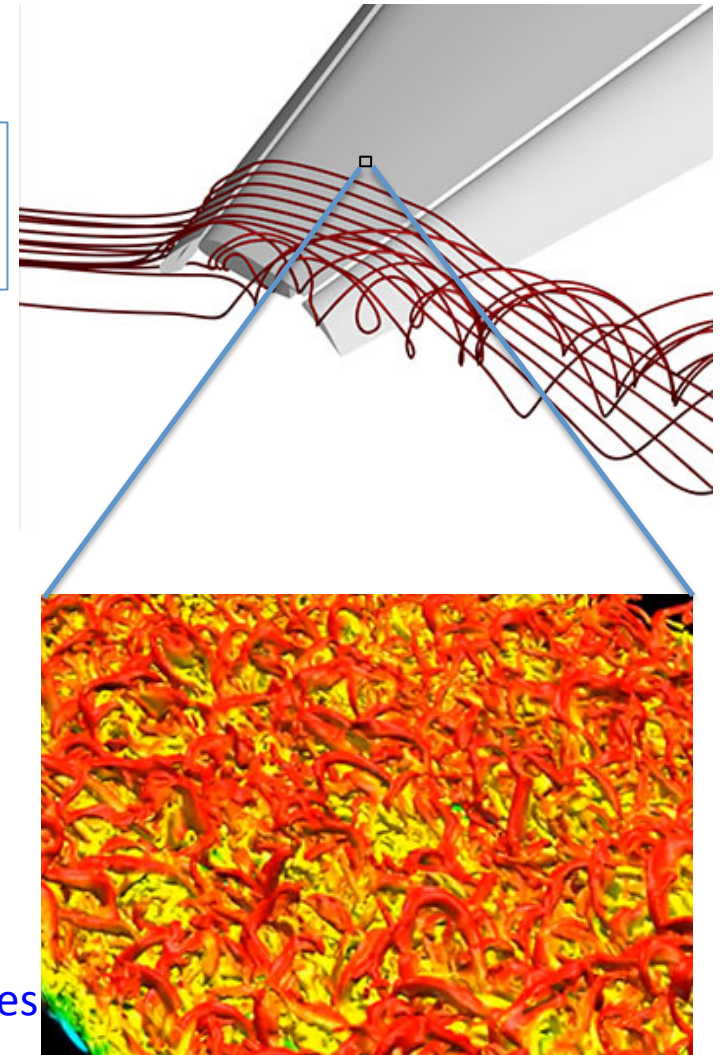


What we cannot compute

Small scale eddies

Closure problem

→ Averaged model can only involve averaged quantities



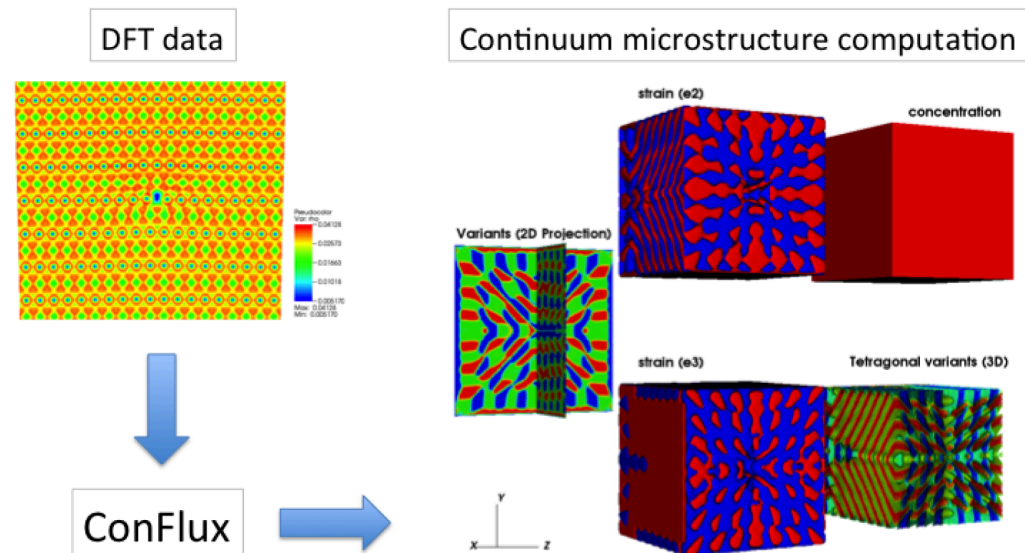
# Materials Modeling

The goal is to identify, explain, predict and ultimately to design the properties and responses of these materials.

Hierarchical models have been developed at several scales

→ These methods have thus far provided insight and qualitative connections to parameters and phenomena from lower scales, but have not been predictive

Quantum Monte Carlo  $\Leftrightarrow$  Density Functional Theory  $\Leftrightarrow$  Continuum physics



With Profs. Vikram Gavini and Krishna Garikipati (Mech Engineering and Materials Science)

# Subject-specific blood flow modeling

Biggest challenges

- ➔ lack of physiologic data to inform the boundary conditions
- ➔ lack of data on mechanical properties of the vascular model

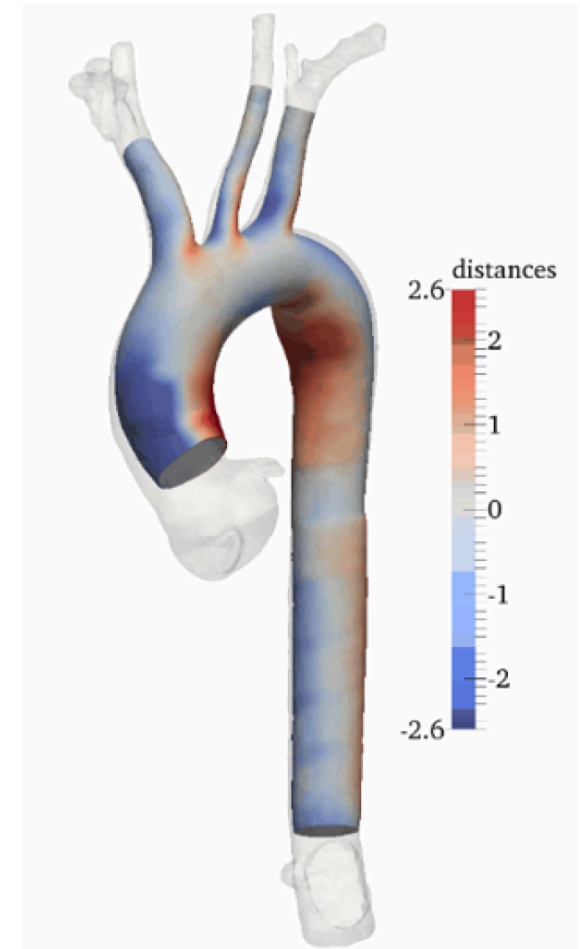
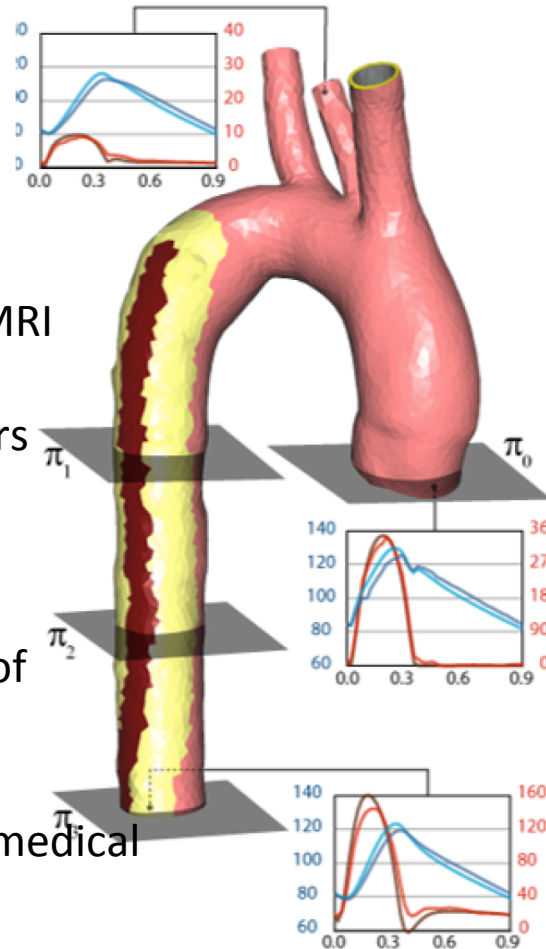
Obtain data from tomography and MRI

Solve inverse problem for parameters

Massive data size

On-the-fly Lagrangian computation of Motion

Evaluation of arterial stiffness from medical Images !



With Prof. Alberto Figueroa (Biomedical Engineering & Surgery)

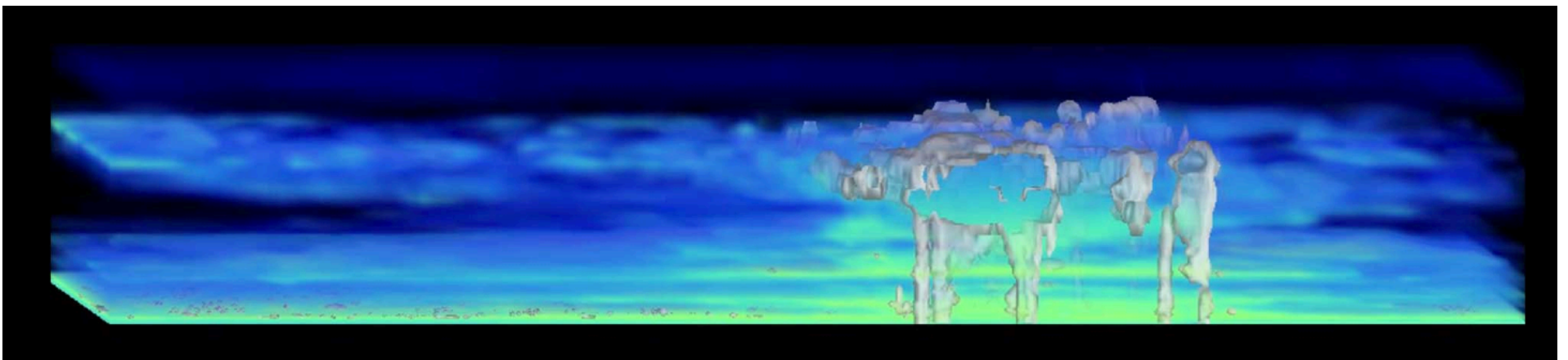
# Climate system interactions

The Earth's climate system is composed of multiple interacting components that span spatial scales of 13 orders of magnitude and temporal scales that range from microseconds to centuries.

➔ key responses and feedbacks in the system are not well characterized

Understanding how clouds interact with the larger scale circulation, thermodynamic state, and radiative balance is one of the most challenging problems

We use statistical inversion and machine learning to explore the interaction between changes in the Earth's climate system and the radiative fluxes, circulation, and precipitation generated by large scale organized cloud systems.



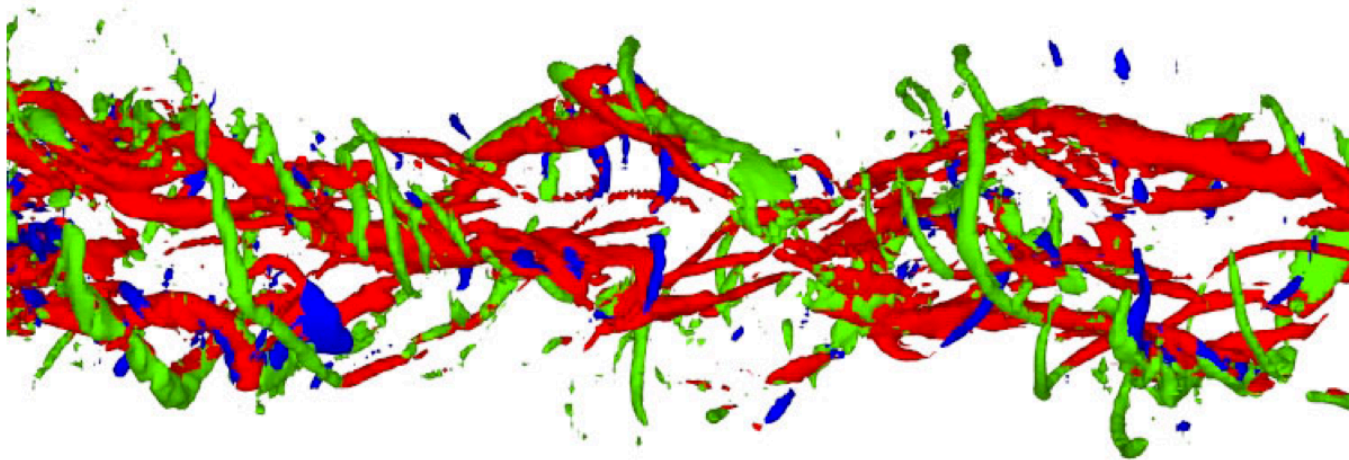
With Prof. Derek Posselt (Atmospheric Oceanic & Space Sciences)

# How can we scale up these problems?

\$3.46M to combine supercomputer simulations with big data

9/3/2015 

From: **Kate McAlpine**  
Michigan Engineering



A new way of computing could lead to immediate advances in aerodynamics, climate science, cosmology, materials science and cardiovascular research. The National Science Foundation is providing \$2.42 million to develop a unique facility for refining complex, physics-based computer models with big data techniques at the University of Michigan, with the university providing an additional \$1.04 million.

