NASA Quantum Computing Workshop

University of Hawaiʻi Mānoa

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Quantum computational methods for fluid dynamics

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November 8, 2017

NASA Workshop on Quantum Computing for Aeroscience and Engineering
Lockheed-Martin Center for Innovation, 8000 Harbour Boulevard, Suffolk, Virginia

Overview

Introduction to quantum computing

Landmark papers leading to quantum computation and quantum simulation

1925: Bose-Einstein Condensate (BEC) prediction [1]

- Discovery of quantum mechanically coherent state of matter

1928: Dirac quantum theory of the electron [2]

- Discovery of noncommuting spin degrees of freedom
- Fundamental description of Fermi matter

1932: Representing spin-f bosons by $2f$ fermions [4]

1935: Einstein-Rosen-Podolsky (EPR) paradox [3]

- Discovery of quantum entanglement

1938: Connection of Bose-Einstein condensation to superfluidity [5] and to superconductivity [6]

1982: Feynman conjecture for simulating physics [7]

- Proposes building a quantum computer for quantum simulation

References

[1] A. Einstein. Quantentheorie des einatomigen idealen gases. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, 1: 3., 1925.

[2] P. A. M. Dirac. The Quantum Theory of the Electron. *Royal Society of London Proceedings Series A*, 117:610–624, February 1928.

[3] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47:777, 1935.

[4] Majorana, E. Atomi orientati in campo magnetico variablile. *Nuovo Cimento*, 9(3):43–50, 1932.

[5] F. London. On the Bose-Einstein condensation. *Phys. Rev.*, 54(11):947–954, 1938.

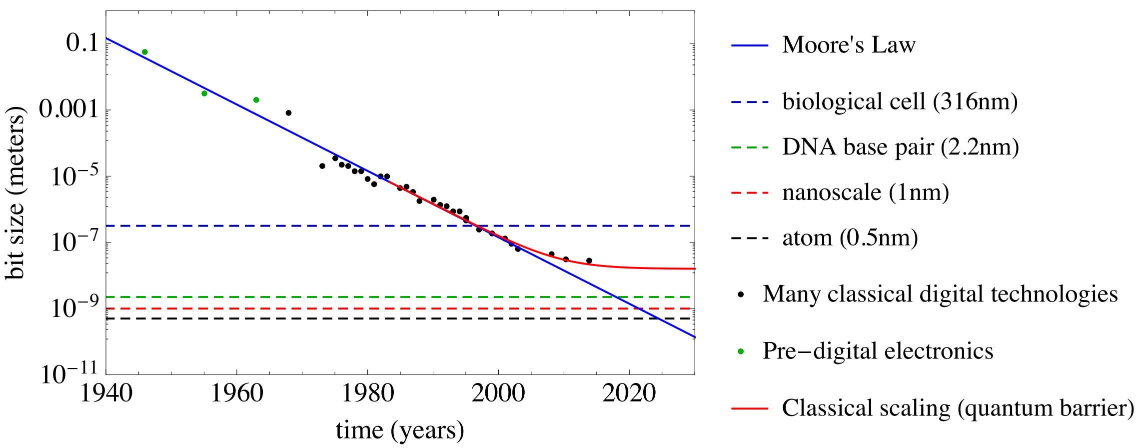
[6] F. London. On the problem of the molecular theory of superconductivity. *Phys. Rev.*, 74:562–573, 1948.

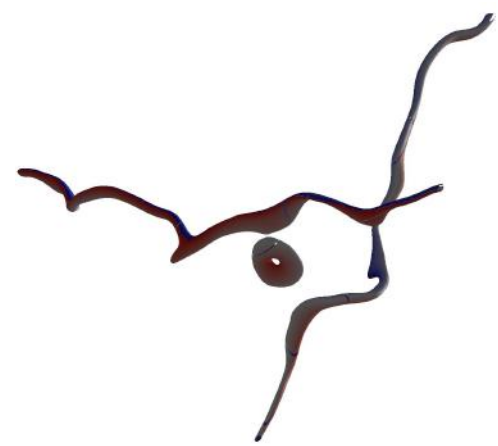
[7] Richard P. Feynman. Simulating physics with computers. *International Journal of Theoretical Physics*, 21(6/7):467–488, 1982.

Quantum computation for quantum simulation

Application areas

- Material science — Exponential Hilbert space of $\sim$1000 qubits (today we have on the order of 5)
- Complex fluid dynamics — Factored Hilbert space of $\sim 10^{24}$ qubits (today we have on the order of 10^5)
  - PDE (such as the Navier-Stokes equation) is only an approximation of macroscopic-scale dynamics
  - It is not practical to write macroscopic-scale equations of motion for dynamics that includes, for example, a combination of multiphase physics, multiple species, shocks, supersonic or relativistic flow, nucleation, chemical reactions, quantum chemistry and ionization physics
  - It is practical to write unitary quantum algorithms that captures all these fundamental physical processes
- Foundations of quantum theory — Understanding and modeling high energy physics and gauge field theories
- It from qubit — Quantum computational models of quantum field theory and gravity





USAF Technical Reports DTIC ADA434366 (1996) [1], ADA421712 (1996) [2] and ADA474659(2007) [3]

Quantum lattice gas papers

Lattice-based quantum algorithms for evolving a spinor field

1946: Feynman checkerboard in 1+1 dimensions [1]
1958: Riazanov lattice algorithm in 3+1 dimensions [2]
1984: Jacobson and Schulman Ising spin chain representation in 1+1 dimensions [3]
1988: 't Hooft deterministic model in 3+1 dimensions [4]
1993: Succi and Benzi quantum lattice Boltzmann model in 3+1 dimensions [5]
1994: Bialynicki-Birula model for the Weyl, Dirac and Maxwell eqs. in 3+1 dimensions [6]
1996: Meyer quantum lattice gas in 1+1 dimensions [7]
2002: Yepez quantum lattice gas in 3+1 dimensions [8]   Lagrangian: $\mathcal{L} = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi$
2016: Yepez quantum lattice gas model for the Dirac-Maxwell-London eqs. in 3+1 dimensions [9]

References

[1] Richard P. Feynman. *California Institute of Technology CIT archives*, (13.3), February 1946.

[2] G.V. Riazanov. *Soviet Physics JETP*, 6 (33)(6):7 pages, June 1958.

[3] Theodore Jacobson and L.S. Schulman. *Journal of Physics A: Math. Gen.*, 17:375–383, 1984.

[4] Gerard 't Hooft. systems. *J. Stat. Phys.*, 53(1-2):323–344, Mar 1988.

[5] Sauro Succi and R. Benzi. *Physica D*, 69:327–332, 1993.

[6] Iwo Bialynicki-Birula. *Physical Review D*, 49(12):6920–6927, 1994.

[7] David A. Meyer. *Journal of Statistical Physics*, 85(5,6):551–574, 1996.

[8] Jeffrey Yepez. *Quantum Information Processing*, 4(6):471–509, Dec 2005 (quant-ph/0210093).

[9] Jeffrey Yepez. *"SPIE Quantum Information Science and Technology II*, 9996N(22):1–20, Oct 2016 (quant-ph/1609.02225).

Overview

Entropic lattice Boltzmann model

Entropic-information model in Q-dimensional kinetic space

- Velocity $\boldsymbol{c}_q$, for $q = 1, 2, \ldots, Q$. Separable dynamics in $\boldsymbol{x}$ and $\boldsymbol{k}$ space

$$f'_q(\boldsymbol{x}) = f_q(\boldsymbol{x}) + \Omega_q(f_1, \ldots f_Q) \qquad f_q(\boldsymbol{k}) = e^{i\boldsymbol{c}_q\cdot\boldsymbol{k}} f'_q(\boldsymbol{k}),$$

  where $f_q = \langle n_q \rangle_{\text{in}}$ and $f'_q = \langle n'_q \rangle_{\text{out}}$ are incoming and outgoing probabilities, respectively.
- Information is conserved via an entropy function

$$H(f_1, \ldots, f_Q) = \sum_q f_q \ln(\gamma_q f_q),$$

  for $\sum_q \gamma_q = 1$, by the constant entropy constraint

$$H(f'_1, \ldots, f'_Q) = H(f_1, \ldots, f_Q).$$

- The hydrodynamic equations for incompressible flow (Navier-Stokes and continuity equations) are respectively replaced with moment equations

$$\sum_q \boldsymbol{c}_q f_q = 0, \quad \text{(NS eq.)} \qquad \sum_q f_q = 0 \quad \text{(continuity eq.)}.$$

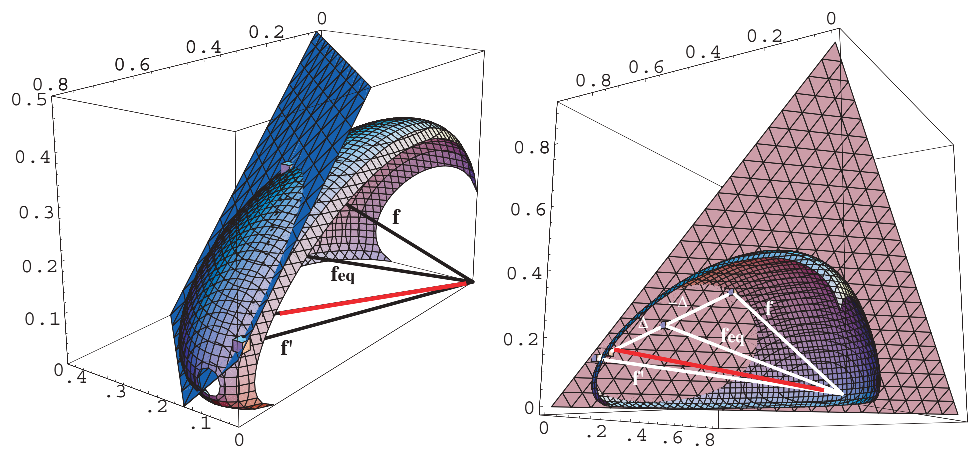
Entropic lattice Boltzmann model

Entropic-information model in Q-dimensional kinetic space

- Entropic lattice Boltzmann equation (discretized kinetic transport) with inverse relaxation time $\alpha\beta$:

$$f_a(\vec{x} + \vec{c}_a \Delta t, t + \Delta t) = f_a(\vec{x}, t) + \alpha\beta \left[f_a^{\text{eq}}(\vec{x}, t) - f_a(\vec{x}, t) \right],$$

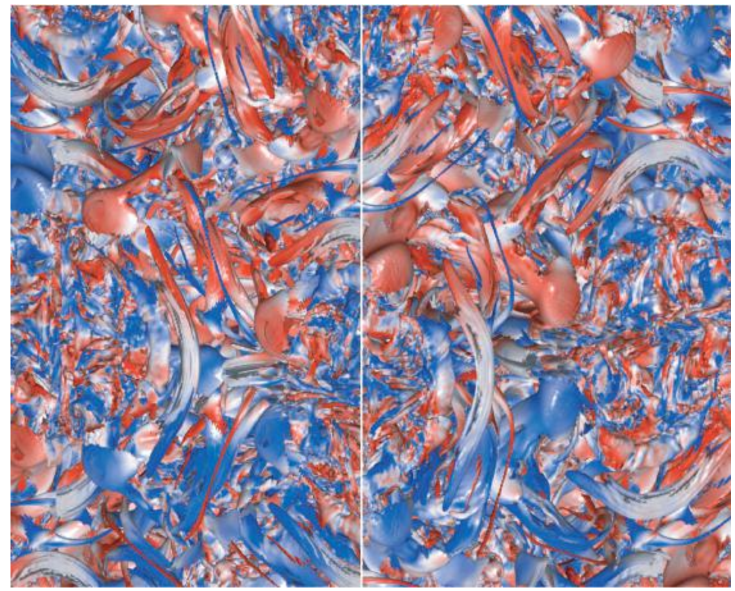
- Boltzmann $\mathcal{H}$-function $\mathcal{H} = \sum_a \Big[\underbrace{f_a \ln(\gamma_a f_a)}_{\text{classical}} + \underbrace{\cancelto{0}{(1 - f_a)\ln(1 - f_a)}}_{\text{quantum}} \Big]$
- The effective shear viscosity is $\nu_{\text{eff}} = c_s^2 \Delta t \left(\frac{1}{\alpha\beta} - \frac{1}{2} \right)$
- Intersection of constant mass plane and constant entropy surface:



Proc. R. Soc. A, 457(2007):717–766 (2001)[4], *PRE*, 68(2):025103 (2003) [5], *Physica D*, 193(1-4):169–181 (2004) [6]
Phil. Trans. R. Soc. Lond. A, 362(1821):1691–1701 (2004)[7]
Euro. Phys. J. Special Topics, 171(1):167–171 (2009)[8], *Physical Review E*, 75(3):036712 (2007) [9]

Classical turbulence

Navier-Stokes turbulence is tangle of vortex tubes

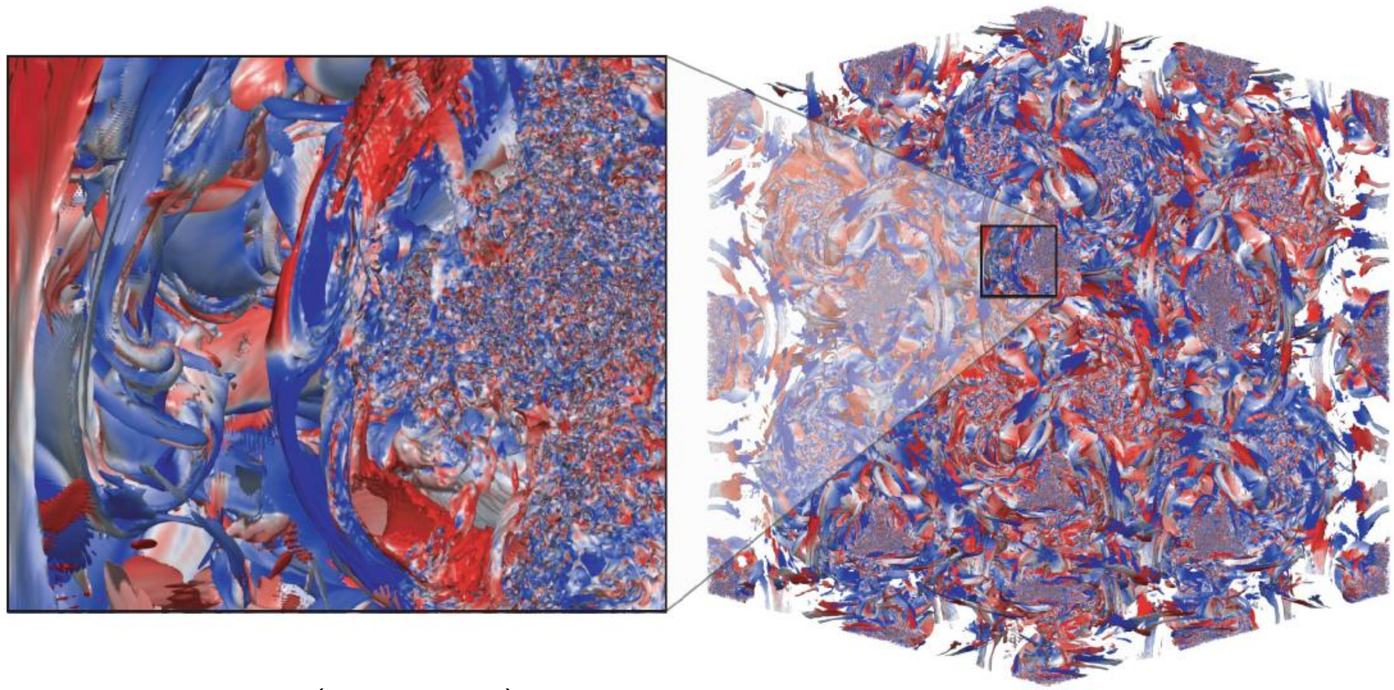


$$\partial_t \boldsymbol{v} + \nabla \cdot (\boldsymbol{v}\boldsymbol{v}) = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \boldsymbol{v}, \qquad \nabla \cdot \boldsymbol{v} = 0.$$

B.M. Boghosian, J. Yepez, P.V. Coveney, and A.J. Wagner, Proceedings of the Royal Society: Mathematical, Physical and Engineering Sciences, A 457 (2001) 717-766.
B.M. Boghosian, P. J. Love, P. V. Coveney, Iliya V. Karlin, S. Succi, J. Yepez, Phys. Rev. E, Vol. 68, 025103(R) (2003).
B. Keating, G. Vahala, J. Yepez, M. Soe, and L. Vahala, Physical Review E, Vol. 75, 036712 (2007).

Navier-Stokes fluid turbulence

NS equation applies above the dissipation scale



- Onset of turbulence with helicity ($\boldsymbol{v} \cdot \nabla \times \boldsymbol{v}$) shown by red to blue coloring.
- Strong coupling exists between large and small scales of incompressible NS flow.
- Ad hoc cutoff of small scale flow is a common problem in classical CFD.
- Quantum computational fluid dynamics does not suffer from a dissipation-scale cutoff problem.

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UNIVERSITY of HAWAI'I MĀNOA

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   - Quantum lattice gas papers
2. **Entropic lattice Boltzmann model**
   - Discrete kinetic space
   - Entropic lattice Boltzmann equation
   - K41 hypothesis and classical turbulence
3. **Time evolution of a quantum gas**
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7. **Future outlooks**

Time evolution of a quantum gas

Classical physics is a special case of quantum physics

- Expectation value of some operator $\hat{\boldsymbol{O}}(x)$ using the system wave function $|\psi_{\boldsymbol{k}}(x)\rangle$ for $x = (t, \boldsymbol{x})$

$$\langle \hat{\boldsymbol{O}}(x) \rangle = \frac{\int dk^D \mathrm{Tr}\left[\boldsymbol{O}|\psi_{\boldsymbol{k}}(x)\rangle\langle\psi_{\boldsymbol{k}}(x)|\right]}{\int dk^D \mathrm{Tr}\left[|\psi_{\boldsymbol{k}}(x)\rangle\langle\psi_{\boldsymbol{k}}(x)|\right]}$$

- Apply the Erhenfest theorem (1927) to a quantum system with Hamiltonian H

$$\frac{d}{dt}\langle \hat{\boldsymbol{O}}(x) \rangle = \frac{1}{i\hbar}\langle [\hat{\boldsymbol{O}}(x), H] \rangle + \frac{\partial}{\partial t}\langle \hat{\boldsymbol{O}}(x) \rangle$$

- Choose momentum operator $\hat{\boldsymbol{p}}$: eq. of motion for and momentum density $\langle \hat{\boldsymbol{p}} \rangle = \rho \boldsymbol{v}$ and pressure $P = g\rho^2/2$

Zero temperature BEC

$$\underbrace{\rho\frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla P - \nabla\left(-\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)}_{\text{Euler-Madelung-Bohm equation for a quantum superfluid}}$$

$$\longrightarrow \qquad \frac{d}{dt}\langle \hat{\boldsymbol{p}} \rangle = \underbrace{\rho\frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla P + \eta\nabla^2\boldsymbol{v}}_{\text{Navier-Stokes equation for a classical viscous fluid}} + \cdots$$

Qubit array encoding

Qubit array encodes both the quantum fields and the spacetime lattice

- With Q qubits per point in the qubit array, the state $|\psi\rangle$ at a point x is a 2^Q-multiplet ket $|\Psi\rangle = \sum_{N=0}^{2^Q-1} \psi_N |N\rangle$
- Binary encoded index is $N = 2^{Q-1} q_1 + 2^{Q-2} q_2 + \cdots + 2q_{Q-1} + q_Q$, for Boolean number variables $q_\alpha = 0, 1$ for $\alpha = 1, 2, \cdots, Q$
- $V = L^3$ number of points on a cubical grid of size L—total number of qubits is VQ
- Point in space is assigned a position-basis ket, $|x, N\rangle$, in a large but finite Hilbert space of size 2^{VQ}
- The position ket $|x, N\rangle = |x, q_1, \cdots, q_Q\rangle$ is the numbered state with all the other numbered variables at points $\neq x$ set to zero

$$|x, q_1, \cdots, q_Q\rangle = |0000 \quad \cdots \underbrace{q_1, q_2, \cdots, q_Q}_{\text{point } x} \cdots \quad 0000\rangle$$

1. Quantum circuit model
2. Measurement-based quantum computation — Example application: Quantum simulation of Navier-Stokes fluidity
3. Adiabatic quantum computation
4. Topological quantum computation
5. Quantum Turing machine
6. Quantum lattice gas model — Example application: Quantum simulation of d-wave superconductivity

Simplest model:

With two qubits per $\boldsymbol{k}$-space point, the fermion configurations at a point in momentum space are encoded as follows:

$$|q_\alpha q_\beta\rangle = \begin{cases} |\psi_0\rangle = |00\rangle_{\boldsymbol{k}} \mapsto \text{empty}, & \text{no fermions}, \\ |\psi_1\rangle = |01\rangle_{\boldsymbol{k}} \mapsto -\boldsymbol{k}\downarrow, & \text{spin-down fermion}, \\ |\psi_2\rangle = |10\rangle_{\boldsymbol{k}} \mapsto \boldsymbol{k}\uparrow, & \text{spin-up fermion}, \\ |\psi_3\rangle = |11\rangle_{\boldsymbol{k}} \mapsto \boldsymbol{k}\uparrow, -\boldsymbol{k}\downarrow, & \text{fermion double}, \end{cases}$$

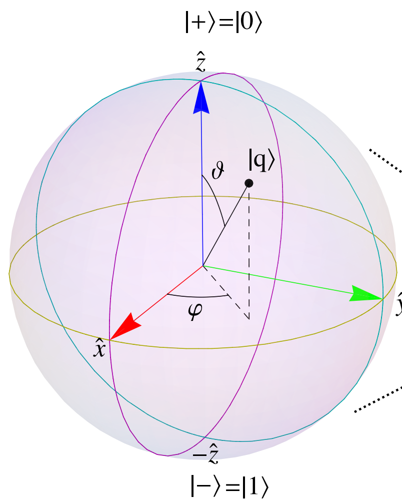
for quantum numbers $\langle\alpha,\beta\rangle = \langle\boldsymbol{k}\uparrow, -\boldsymbol{k}\downarrow\rangle$. The qubit ordering is fixed; that is, a double occupancy is encoded in only one way as $\uparrow\downarrow$.

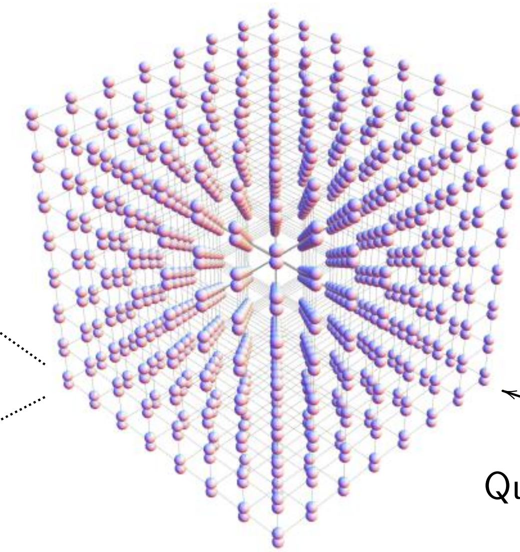
Quantum lattice gas paradigm

Qubit array: Spinor $\psi(x)$ (e.g. 4-amplitudes with 2 qubits/point)

- Initial separable quantum state: $\Psi(t) = \bigotimes_{\boldsymbol{x}} \psi(t, \boldsymbol{x})$ Full Hamiltonian: $\boldsymbol{H}$
- Quantum simulation representations:
  - Unitary form (digital): $\Psi(t+\tau) = U\Psi(t)$ with a $U = e^{-i\frac{\boldsymbol{H}\ell}{\hbar c}} \cong \bigotimes_{\boldsymbol{x}} e^{-i\frac{\hat{\boldsymbol{h}}(\boldsymbol{x})\ell}{\hbar c}}$
  - Hermitian form (analog): $i\hbar\partial_t\psi(x) = \hat{\boldsymbol{h}}(x)\psi(x)$ with local Hamiltonian $\hat{\boldsymbol{h}}(x)$
- $|\psi(x)\rangle = \underbrace{\left[\Psi^+_{\|}(x)|00\rangle + \Psi^-_{\|}(x)|11\rangle\right]}_{\text{entangled Cooper pair}} + \underbrace{\left[\Phi^+_{\perp}(x)|01\rangle + \Phi^-_{\perp}(x)|10\rangle\right]}_{\text{nonseparable particle-hole}} = |\Psi_{\|}(x)\rangle + |\Phi_{\perp}(x)\rangle$

Bloch sphere





Quantum gate acts on a qubit EPR pair

- Local in $\boldsymbol{k}$-space
- BEC superfluid with dimers

Quantum state of a quantum lattice gas

Grid: $d = 1 + D$ dimensional space of discrete points $x^\mu = (t, \boldsymbol{x}) \in \mathbb{Z}^d$

- **For grid of size L, volume of space: L^d**
- **State of the system of particles at time t:**
  - Tensor product state over entangled cluster states $|\psi(x)\rangle$

$$|\Psi(t)\rangle \xrightarrow[\text{separated}]{\text{naturally}} \bigotimes_{\boldsymbol{x}=1}^{L^d} |\psi(t, \boldsymbol{x})\rangle$$

  - Globally entangled state

$$|\Psi(t)\rangle = \sum_{N=0}^{2^{2L^d}-1} \Psi(t)|N\rangle$$

- **Unitary evolution $|\Psi(t + \delta t)\rangle = U|\Psi(t)\rangle$**
  - Quantum lattice gas is a generalization of a lattice gas model with bits replaced by qubits
- **Quantum lattice gas dynamics**
  - A particle at point x^μ encoded by a single qubit with number variable $n_k = 0, 1$ and probability $f_k = \langle n_k \rangle$
  - Quantum lattice Boltzmann equation: $f_k(x^\mu + \ell e^\mu) = f_k(x^\mu) + \Omega_k[f_*(x^\mu)]$

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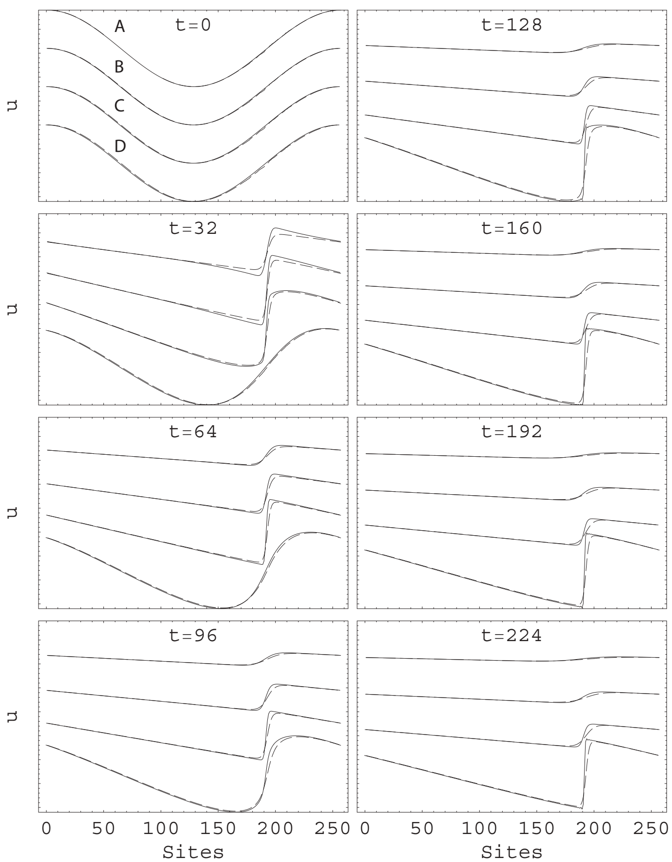
Burgers equation in 1+1 dimensions

Mesh architecture using quantum mechanical design principles



$$\begin{pmatrix} a \\ b \end{pmatrix} \to \begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \langle q_\alpha q_\beta | n^+_{\alpha\beta} | q_\alpha q_\beta \rangle \\ \langle q_\alpha q_\beta | n^-_{\alpha\beta} | q_\alpha q_\beta \rangle \end{pmatrix}$$

$$n^+_{\alpha\beta} = U^\dagger n_\beta U \quad \text{and} \quad n^-_{\alpha\beta} = U^\dagger n_\alpha U$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} + \frac{i}{2} & -\frac{1}{2} - \frac{i}{2} & 0 \\ 0 & \frac{1}{2} + \frac{i}{2} & \frac{1}{2} + \frac{i}{2} & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$$

Derived: $\mathcal{H} = -\sum_{s=\pm} \left[\underbrace{f_s \ln(\gamma_s f_s)}_{\text{classical}} + \underbrace{(1 - f_s)\ln(1 - f_s)}_{\text{quantum}} \right]$

Shock formation is an emergent phenomenon—not coded by a PDE!

(A) $\nu = 8$, (B) $\nu = 2$, (C) $\nu = \frac{1}{2}$, (D) $\nu = \frac{1}{32}$

Theory: *Journal of Statistical Physics*, 107(1):203–224 (2002) [10], *Physical Review A*, 74(4):042322 (2006) [11]
Experiment (NMR QIP): *Physical Review A*, 74(4):042321 (2006) [12]

Micro-, meso- and macroscopic scales

Quantum lattice gas model's spatial scales

- Microscopic scale
  - discrete spacetime — band-limited description
  - unitary operator-centric quantum mechanical dynamics
  - alternative to Hamiltonian-centric dynamics commonly used in AMO, condensed matter and solid-state physics
  - quantum spin system on a lattice — discrete version of the many-body Schrodinger equation
- Mesoscopic scale
  - discrete spacetime
  - **quantum lattice Boltzmann equation** for kinetic transport (exact description)
- Macroscopic scale
  - spacetime continuum (Bloch-Wannier picture)
  - set of partial differential equations of motion (effective Navier-Stokes hydrodynamics as small $\boldsymbol{k}$)
  - continuous fields (e.g. number density, momentum density, energy density)

- **Microscopic dynamics**

  Quantum mechanical wave equation: $|\Psi(t+\tau)\rangle = e^{i\hat{H}\tau/\hbar}|\Psi(t)\rangle = \hat{S}\hat{C}|\Psi(t)\rangle$. Collision matrix $\hat{C} = \bigotimes_{x=1}^{V}\hat{U}$ acts on each lattice node independently and causing quantum entanglement of the outgoing configurations of the particles

- **Mesoscopic dynamics**

  Quantum lattice Boltzmann equation:

$$f_a(\boldsymbol{x} + \ell\hat{e}_a, t+\tau) = f_a(\boldsymbol{x}, t) + \mathrm{Tr}\left[\varrho(t)(\hat{C}^\dagger\hat{n}_\alpha\hat{C} - \hat{n}_\alpha)\right]$$

- **Macoscopic dynamics** (long-wavelength, low-frequency, and subsonic limits)

  Fermi-Dirac distribution: $f_a^{\mathrm{eq}} = \frac{1}{\frac{1}{\gamma_a}e^{\beta E_a}+1} = \frac{1}{e^{\beta(E_a+\Delta E_a)}+1}$

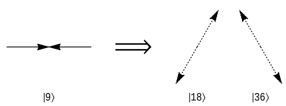
  Chapman-Enskog expansion with local decoherence leads to a continuity equation and momentum equation:

$$\begin{aligned} \partial_t\rho + \partial_i(\rho v_i) &= 0 \\ \partial_t(\rho v_i) + \partial_j(g\rho v_i v_j) &= -\partial_i P + \eta\partial^2 v_i + \left(\zeta + \frac{\eta}{D}\right)\partial_i\partial_j v_j + \cdots \end{aligned}$$

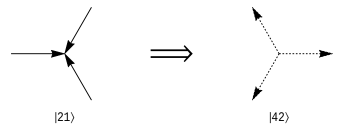
Particle-particle collisions

Measurement-based quantum lattice gas model of fluid dynamics in 2+1 dimensions

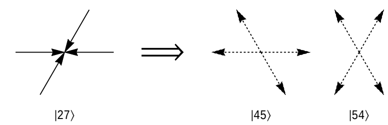
Examples of particle-particle collisions in a classical lattice gas:



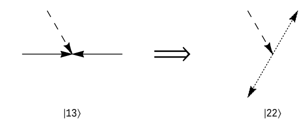
(a) a 2-body state $|9\rangle$ transitions to state $|18\rangle$ or $|36\rangle$



(b) a 3-body state transition, and the reverse transition occurs too

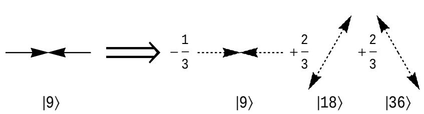


(c) a 4-body state $|27\rangle$ transitions to state $|45\rangle$ or $|54\rangle$

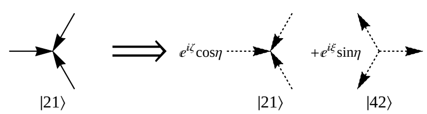


(d) 2-body state transition with a spectator particle (dashed line)

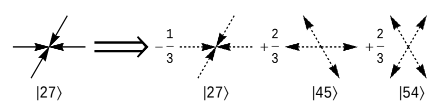
Examples of particle-particle collisions in a quantum lattice gas:



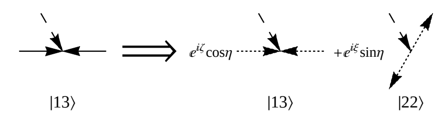
(e) a 2-body state $|9\rangle$ transitions to the entangled cluster state $-\frac{1}{3}|9\rangle + \frac{2}{3}|18\rangle + \frac{2}{3}|36\rangle$



(f) a 3-body state $|21\rangle$ transitions to the entangled cluster state $e^{i\zeta}\cos\eta|21\rangle + e^{i\xi}\sin\eta|42\rangle$



(g) a 4-body state $|27\rangle$ transitions to the entangled cluster state $-\frac{1}{3}|27\rangle + \frac{2}{3}|45\rangle + \frac{2}{3}|54\rangle$

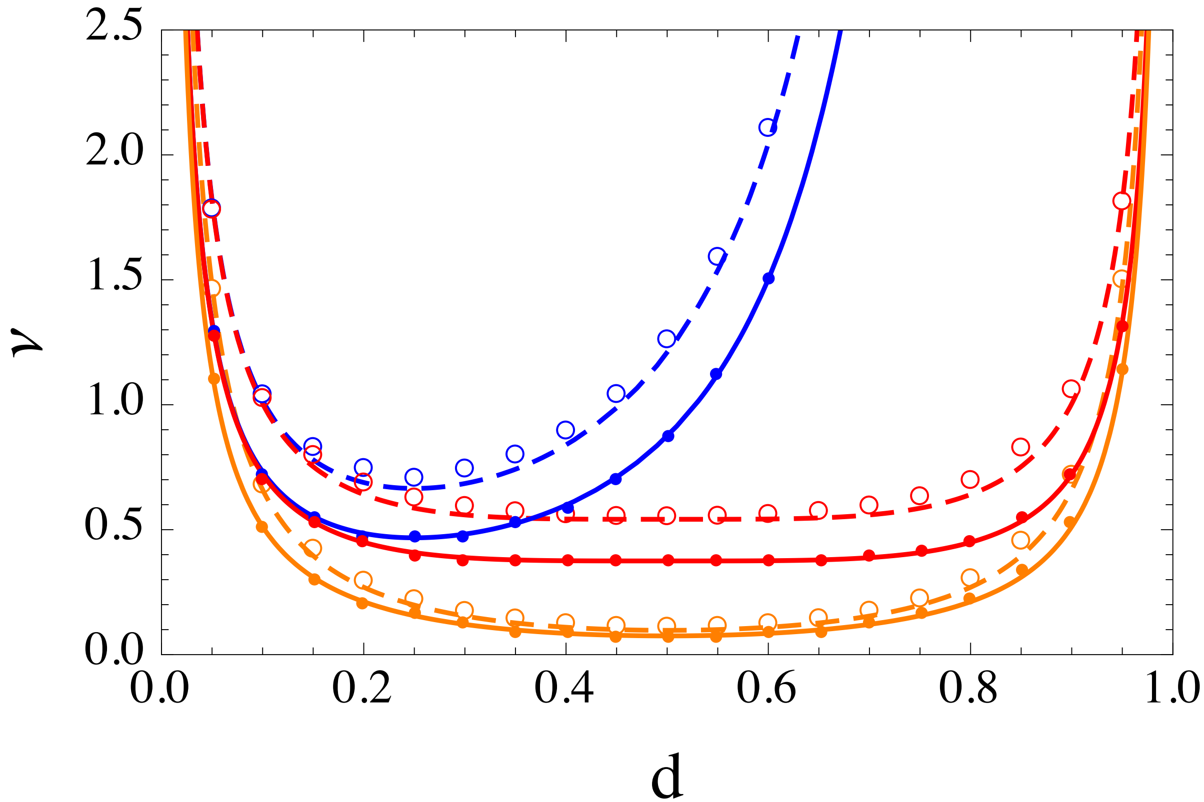


(h) state $|13\rangle$, 2-body with a spectator particle (dashed line), transitions to entangled cluster state $e^{i\zeta}\cos\eta|13\rangle + e^{i\xi}\sin\eta|22\rangle$

Kinematic shear viscosity

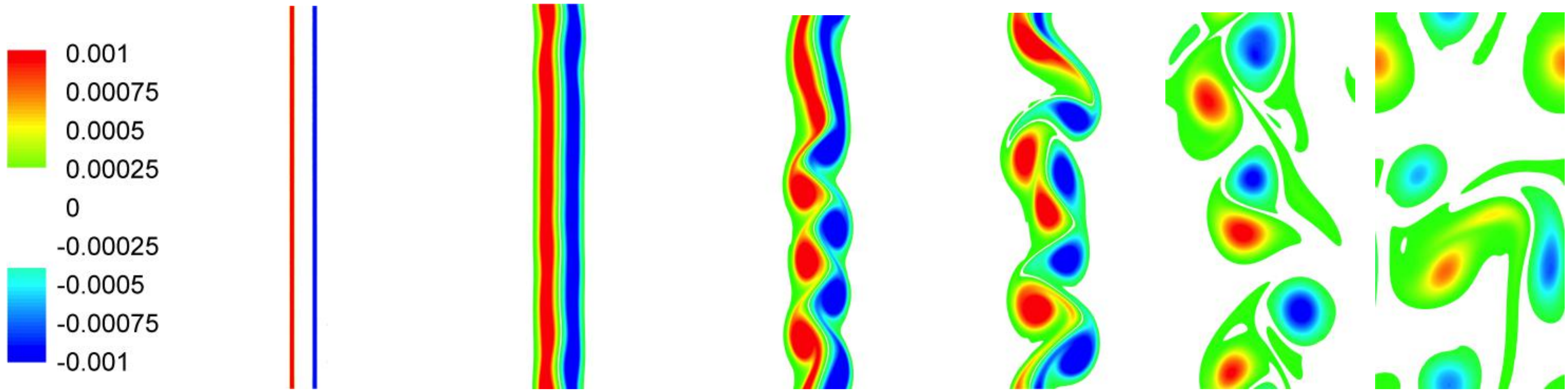
Quantum lattice Boltzmann equation theory versus quantum simulation

The computed viscosities ν as a function of density (circle=classical and dot=quantum) match the values given by the theory of the quantum lattice Boltzmann equation (solid curves) over the entire range of densities $d \in [0, 1]$



Navier-Stokes flow

Quantum simulation of the Kelvin-Helmholtz instability



(i) $|\nabla \times \boldsymbol{v}|$ (j) $t = 0$ (k) $t = 100K$ (l) $t = 150K$ (m) $t = 200K$ (n) $t = 400K$ (o) $t = 700K$

Contour plot of vorticity at initialization. Center flow is upward at a velocity of 0.28, and outer flow is downward at a velocity of 0.04 with a background density of $d = 0.17$. Start of the KH instability occurs around 100,000 time steps. By 150,000 time steps, the KH instability causes the breakup of the shear layer into separate vortex pairs. By 400,000 time steps, dissipation begins to dominate the flow. By 700,000 time steps merging of the vortices has started.

Quantum Computing and Quantum Communications, pages 34–60. Lecture Notes in Computer Science, Springer-Verlag (1999) [13], *Physical Review E*, 63(4):046702 (2001)[14] *Phys. Rev. E*, 92:033302 (2015) [15]

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Nonlinear quantum gas models

Scalar Bose-Einstein condensate

- Nonrelativistic composite bosons (pairs of fermions) with an interaction Lagrangian density $\mathcal{L}_{\text{int}} \propto \varphi^4$

- The time-dependent theory for a nonrelativistic condensate in the zero-temperature limit in free space is

$$\mathcal{L}_{\text{BEC}}(t) = i\hbar\,\varphi^*\partial_t\varphi + \frac{\hbar^2}{2m}(\nabla\varphi^*)\cdot\nabla\varphi + \mu\,\varphi^*\varphi - \varphi^* V_{\text{H}}\,\varphi,$$

where $\varphi(x)$ is a complex scalar field for the Bose-Einstein condensate, V_{H} is a local self-consistent Hartree potential

- Effective model for superfluid dynamics

$$i\hbar\partial_t\varphi = -\frac{\hbar^2}{2m}\nabla^2\varphi + \left(g|\varphi|^2 - \mu\right)\varphi\,,$$

where g is coupling strength and μ is the chemical potential.

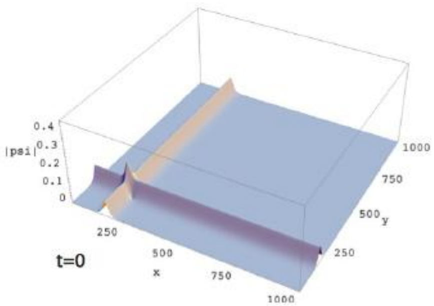
- Bose-Einstein condensate, Helium II phase of ^{4}He below 100 mK, quark-matter condensates in neutron stars, and a Q2 quantum lattice gas.
- Linear quantum flow in the bulk and nonlinear near a topological defect, *i.e.* a quantum vortex
- Vortex-vortex interactions and quantum Kelvin wave-sound waves interactions

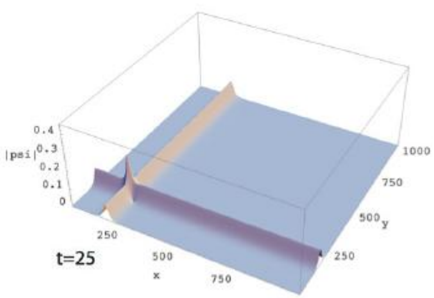
Gross, *J.Math.Phys.*, 4(2):195–207 (1963) [16], Pitaevskii, *Soviet Phys. JETP*, 13(2):451–454 (1961) [17]

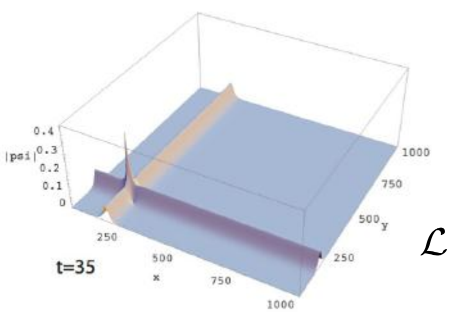
Scalar BEC from a Fermi condensate

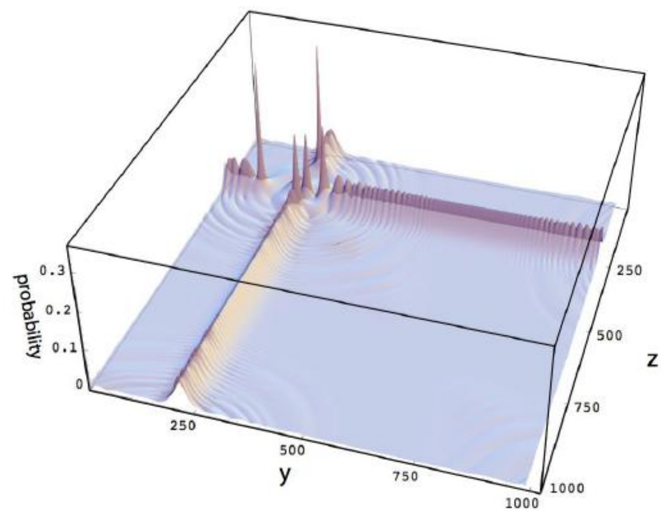
Evolution of a two orthogonally oriented 1D soliton wave trains

- orthogonally directed 1D soliton wave trains
- rapid instability is immediately triggered
- rising peak at the intersection center









$$\mathcal{L}_{\mathsf{NR}} = \sum_{s=\mathsf{L,R}} \left[i\hbar\,\psi_s^\dagger\, \partial_t \psi_s + \frac{\hbar^2}{2m}\psi_s^\dagger\, \sigma_x \nabla^2\psi_s + \psi_s^\dagger\, \left(mc^2 - V_{\mathsf{H}}(\rho)\right)\sigma_x\psi_s\right]$$

Quantum Information Processing, 4(6):457–469 (2005) [18]

Scalar BEC from a Fermi condensate

Nonlinear quantum lattice gas model

- half a trillion qubits
- 20 trillion gates
- 80,000 cycle times
- 10^{18} gate operations
- Einstein CRAY XT-5
- $\varphi = \psi_{\mathsf{L}} + \psi_{\mathsf{R}}$

$$\mathcal{L}_{\mathsf{NR}} = \sum_{s=\mathsf{L},\mathsf{R}} \left[i\hbar\, \psi_s^\dagger\, \partial_t \psi_s + \frac{\hbar^2}{2m} \psi_s^\dagger\, \sigma_x \nabla^2 \psi_s + \psi_s^\dagger\, \Big(mc^2 - V_{\mathsf{H}}(\rho) \Big)\, \sigma_x \psi_s \right]$$

Physical Review Letters, 103(8):084501 (2009) [19]

Kolmogorov turbulence

K41 hypothesis

• Kolmogorov law for incompressible kinetic energy spectrum $E(k)$:

$$E(k) = C\,\varepsilon^{\frac{2}{3}}\,k^{-\frac{5}{3}}$$

• The flow velocity, the kinematic viscosity, and the viscous dissipation quantities have dimensions: $[u] = \frac{L}{T}$, $[\nu] = \frac{L^2}{T}$, and $[\varepsilon] = \frac{L^2}{T^3}$

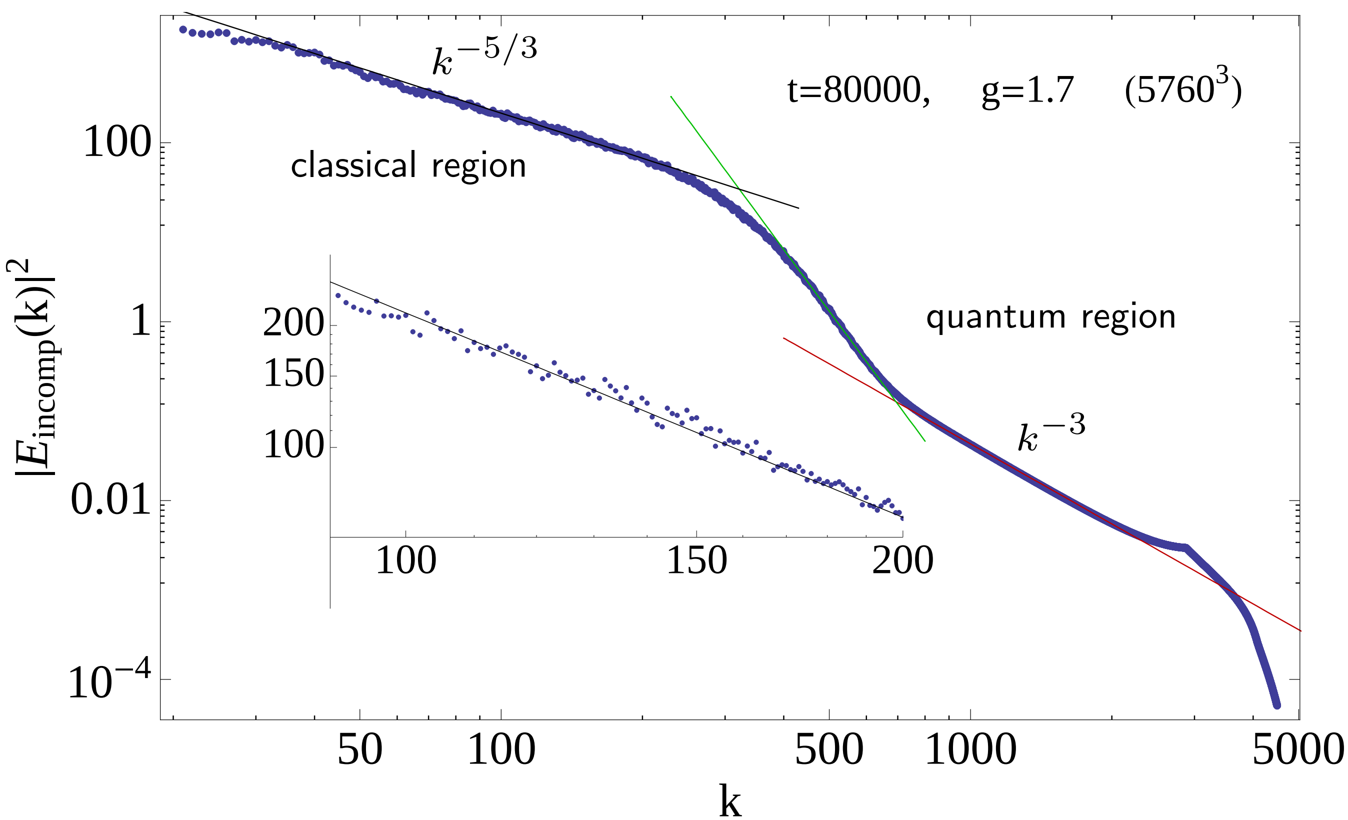
• Dissipation scale $\lambda \equiv \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$ is the smallest classical hydrodynamic scale

• Classical Kolmogorov wave number $k_c = \left(\frac{\varepsilon}{\nu^3}\right)^{\frac{1}{4}}$ (inverse of the dissipation scale) marks the end of the $k^{-5/3}$ inertial subrange

• At wave numbers $> k_c$, kinetic energy is ultimately dissipated at a rate ε.

Energy spectrum on 5760^3: Kolmogorov turbulence

Quantum to classical transition
from a viscosity-free BEC superfluid to a viscous Navier-Stokes fluid



J. Yepez, G. Vahala, L. Vahala, and M. Soe, Physical Review Letters 90, 067902 (2009)

Spinor Bose-Einstein condensates

Nonlinear quantum lattice gas model

- Lagrangian density for a spin-2 BEC

$$\mathcal{L}_{f=2} = i\hbar\,\hat{\psi}^\dagger\,\partial_t\hat{\psi} + \frac{\hbar^2}{2m}\hat{\psi}^\dagger\,\mathbf{1}_{2f+1}\otimes\left[\sigma_x\nabla^2\hat{\psi} + \sigma_x\left(mc^2 - g_0\hat{\varphi}^\dagger\hat{\varphi}\right)\right]\hat{\psi}$$
$$- \hat{\psi}^\dagger\,\frac{g_1}{\hbar^2}\left(\hat{\boldsymbol{F}}.\boldsymbol{f}\right)\otimes\mathbf{1}\hat{\psi} - \hat{\psi}^\dagger\,\frac{g_2}{2}\left(N_{00}\hat{\varphi}^\dagger\hat{\varphi}N_{00}\right)\otimes\mathbf{1}\hat{\psi}$$

- Bosonic field operator $\hat{\varphi} = \hat{\psi}_{\mathsf{L}} + \hat{\psi}_{\mathsf{R}}$
- Equation of motion

$$i\hbar\partial_t\hat{\psi} = \mathbf{1}_{2f+1}\otimes\underbrace{\sigma_x\left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + g_0\hat{\varphi}^\dagger\hat{\varphi}\right)}_{\text{diagonal SU(2) part}}\hat{\psi} + \underbrace{\left[\frac{g_1}{\hbar^2}\left(\hat{\boldsymbol{F}}.\boldsymbol{f}\right) + \frac{g_2}{2}\left(N_{00}\hat{\varphi}^\dagger\hat{\varphi}N_{00}\right)\right]\otimes\mathbf{1}}_{\text{nondiagonal SO(5) part}}\,\hat{\psi}$$

- Operator splitting is exact because of anticommutation relation

$$\left[\frac{g_1(\hat{\boldsymbol{F}}\cdot\boldsymbol{f})\otimes\mathbf{1}}{\hbar^2} + \frac{g_2\hat{N}^{00}\otimes\mathbf{1}}{2}, \mathbf{1}_{2f+1}\otimes\sigma_x\ell^2\nabla^2\right] = 0$$

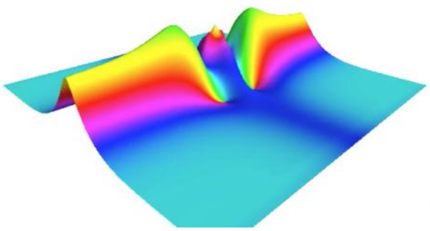
- Quantum algorithm: $\hat{\psi}(\boldsymbol{x}, t+\tau) = U^{\mathsf{nd}}_{f=2}[\hat{\psi}]U^{\mathsf{d}}[\hat{\psi}]\,\hat{\psi}(\boldsymbol{x}, t)$

arXiv:1609.02229 [cond-mat.quant-gas] (2016) [20]

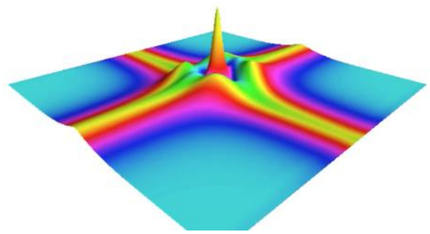
University of Hawaiʻi Mānoa

Spinor BEC from a non-Abelian Fermi condensate

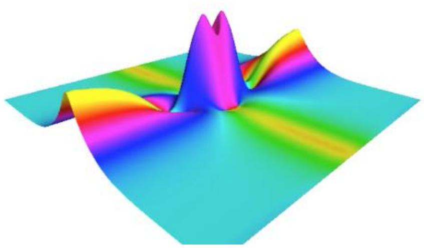
Evolution of a two orthogonally oriented 1D soliton wave trains



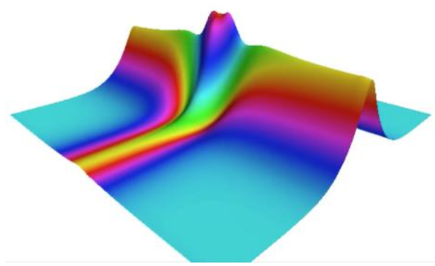
$f = 2, m_f = 0$



$f = 2, m_f = 1$



$f = 2, m_f = -2$



$f = 2, m_f = 2$

- orthogonally directed 1D soliton wave trains
- rapid instability is triggered on all Zeeman levels
- oscillating and rising peaks at the intersection center

$$\mathcal{L}_{f=2} = i\hbar\, \hat{\psi}^\dagger\, \partial_t \hat{\psi} + \tfrac{\hbar^2}{2m} \hat{\psi}^\dagger\, \mathbf{1}_{2f+1} \otimes \left[\sigma_x \nabla^2 \hat{\psi} + \sigma_x \left(mc^2 - g_0 \hat{\varphi}^\dagger \hat{\varphi} \right) \right] \hat{\psi}$$

$$- \hat{\psi}^\dagger\, \tfrac{g_1}{\hbar^2} \left(\hat{\boldsymbol{F}} . \boldsymbol{f} \right) \otimes \mathbf{1} \hat{\psi} - \hat{\psi}^\dagger\, \tfrac{g_2}{2} \left(N_{00} \hat{\varphi}^\dagger \hat{\varphi} N_{00} \right) \otimes \mathbf{1} \hat{\psi}$$

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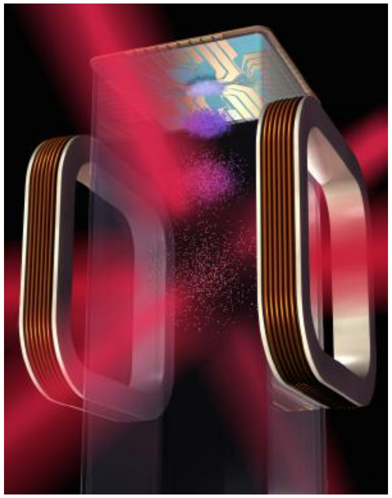
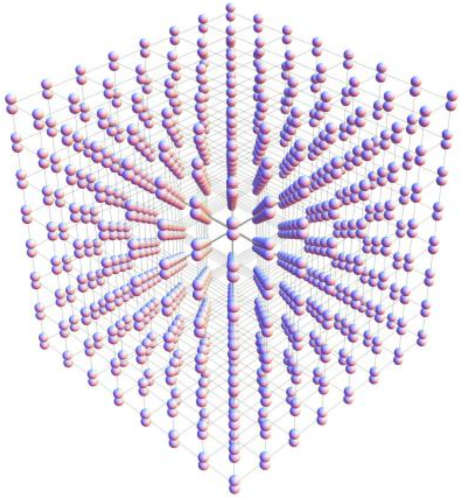
Ultracold quantum gas Bose-Einstein condensate

- 2 ColdQuanta Table-Top Laser-Cooled BEC Systems are installed at the Joint AFRL-UH QC Lab
- Quantum lattice gas experimental representation:
  - Hermitian form (analog QC): $i\hbar\partial_t\psi(x) = h\psi(x)$ with an engineered local Hamiltonian h





- Loading from magneto optical trap (MOT), to a compressed MOT, to a dimple trap below the atom chip
- Air Force takes table-top approach to quantum physics: http://www.wpafb.af.mil/news/story.asp?id=123429620

- Model solid-state condensed matter systems
- Spinor BEC superfluid on a lattice with quantum tunneling between lattice points
- Local qubit pair (Zeeman manifold) at each lattice point

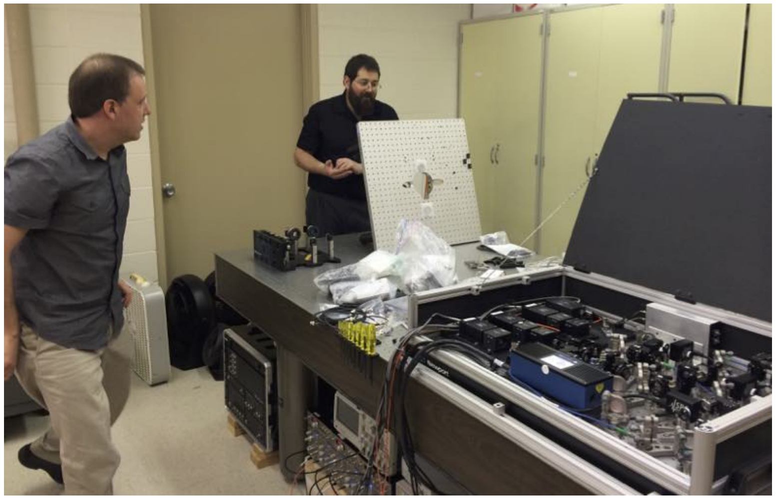
Ultracold quantum gas in an optical lattice

Tech Transition from UH Manoa to AFRL/Space Vehicles & NASA Cold Atom Lab on ISS

UNIVERSITY of HAWAI'I MĀNOA

2nd generation BEC Cart prototype
RAs: Ryan Tobin and Jasper Taylor

Quantum Optics Laser Subsystem and
Dr. Dan Farkas & Evan Salim, Cold Quanta Inc.



BEC Lab in 2016 in Watanabe Hall

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Dirac-Maxwell-London equations

Relativistic superconducting fluid

- The Dirac-Maxwell equations are closely related to the equations of motion for a superconducting fluid comprised of entangled pairs of electrons (Cooper pairs) in the superfluid phase of Bose-Einstein condensate
  - Connection between Bose-Einstein condensation and superfluidity and superconductivity was originally discovered by London London, *Phys. Rev.*, 54(11):947–954 (1938) [21], London, *Phys. Rev.*, 74:562–573 (1948) [22]
- The equations of motion are the coupled Dirac-Maxwell-London equations obtained by relating the charge 4-current density to the 4-potential according to the identity $\lambda_L^2 e J^\mu = -A^\mu$

$$
\begin{aligned}
i\hbar c\,\gamma_\mu \left(\partial^\mu - i\frac{eA^\mu}{\hbar c}\right)\psi &= mc^2\psi \\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu \\
-\frac{1}{\lambda_L^2}A^\nu &= \partial_\mu F^{\mu\nu} \qquad \text{(Maxwell-London equations)}
\end{aligned}
$$

- What Lagrangian density gives the Dirac-Maxwell-London equations?

Landau-Ginzburg field theory

Phenomenological bosonic model of superconductivity

- Lagrangian density functional for a Landau-Ginzburg bosonic field ϕ interacting with a Maxwell field A^μ is

$$\mathcal{L}_{\text{Landau-Ginzburg}} = \hbar c \left|\mathcal{D}_\mu \phi\right|^2 - \frac{1}{4}\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)^2 - V(\phi)$$

  where there is minimal coupling between the ϕ and A^μ fields via $\mathcal{D}_\mu = \partial_\mu + i\frac{e}{\hbar c}A_\mu(x)$, and there is a nonlinear self-coupling for the ϕ field via $\mathcal{L}_{\text{nonlinear}}[\phi] = -V(\phi) = \mu^2|\phi|^2 - \frac{\lambda}{2}|\phi|^2$

- Theory of a type-II superconductor, i.e. a superconducting fluid with magnetic quantum vortices
- The Maxwell field A_μ acquires a mass (say m_{L} via the Higgs mechanism), so the A_μ field can penetrate into a superconductor only up to a depth of m_{L}^{-1}—i.e. the Meissner effect
- Landau-Ginzberg Lagrangian density for the bosonic Cooper pair field is also known as the Abelian Higgs model
- The dual theory of the Abelian Higgs model should be a many-fermion Lagrangian density!
- So what is the many-fermion Lagrangian density?

Quantum lattice gas algorithm for superconductivity

Recast the Maxwell equations using strictly 4-spinor fields

- Introduce 4-spinor potential field $\mathcal{A}$, 4-spinor electromagnetic field $\mathcal{F}$, and 4-spinor current density field $\mathcal{J}$, respectively

$$\mathcal{A} = \frac{1}{\sqrt{2}}\begin{pmatrix} -A_x + iA_y \\ A_0 + A_z \\ -A_0 + A_z \\ A_x + iA_y \end{pmatrix} \qquad \mathcal{F} = \frac{1}{\sqrt{2}}\begin{pmatrix} -F_x + iF_y \\ -\partial \cdot A + F_z \\ \partial \cdot A + F_z \\ F_x + iF_y \end{pmatrix} \qquad \mathcal{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} -J_x + iJ_y \\ \rho + J_z \\ -\rho + J_z \\ J_x + iJ_y \end{pmatrix},$$

where $\boldsymbol{F} = -\partial_0 \boldsymbol{A} - \nabla A_0 + i\nabla \times \boldsymbol{A}$

- Maxwell equations can be written using tensor-product notation

$$\begin{aligned} \mathcal{F} + \mathbf{1} \otimes \bar{\sigma} \cdot \partial \mathcal{A} &= 0 \\ e\mathcal{J} + \mathbf{1} \otimes \sigma \cdot \partial \mathcal{F} &= 0 \end{aligned}$$

where $\sigma^\mu = (1, \boldsymbol{\sigma})$, $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$, $\sigma \cdot \partial = \sigma^\mu \partial_\mu$, and $\bar{\sigma} \cdot \partial = \bar{\sigma}^\mu \partial_\mu$

SPIE Quantum Information Science and Technology II, 9996(22), (2016) [23]

Fermionic version of London superconductivity

Maxwell-London equations expressed using strictly 4-spinor fields

- Complex electromagnetic 4-vector $F^{\mu} = (F_0, \boldsymbol{F}) = (-\partial_\nu A^\nu, -\partial_0 \boldsymbol{A} - \nabla A_0 + i\nabla \times \boldsymbol{A})$
- Consider the 4-spinor potential field $\mathcal{A}$ and dual 4-spinor field $\tilde{\mathcal{A}}$

$$\mathcal{A} = \frac{1}{\sqrt{2}}\begin{pmatrix} -A_x + iA_y \\ A_0 + A_z \\ -A_0 + A_z \\ A_x + iA_y \end{pmatrix} \quad \text{and} \quad \tilde{\mathcal{A}} = -i\lambda_L \frac{1}{\sqrt{2}}\begin{pmatrix} -F_x + iF_y \\ -\partial \cdot A + F_z \\ \partial \cdot A + F_z \\ F_x + iF_y \end{pmatrix}$$

- With the London identity in 4-spinor form $e\mathcal{J} = -\mathcal{A}/\lambda_L^2$, the Dirac-Maxwell-London equations can be written in a symmetrical way

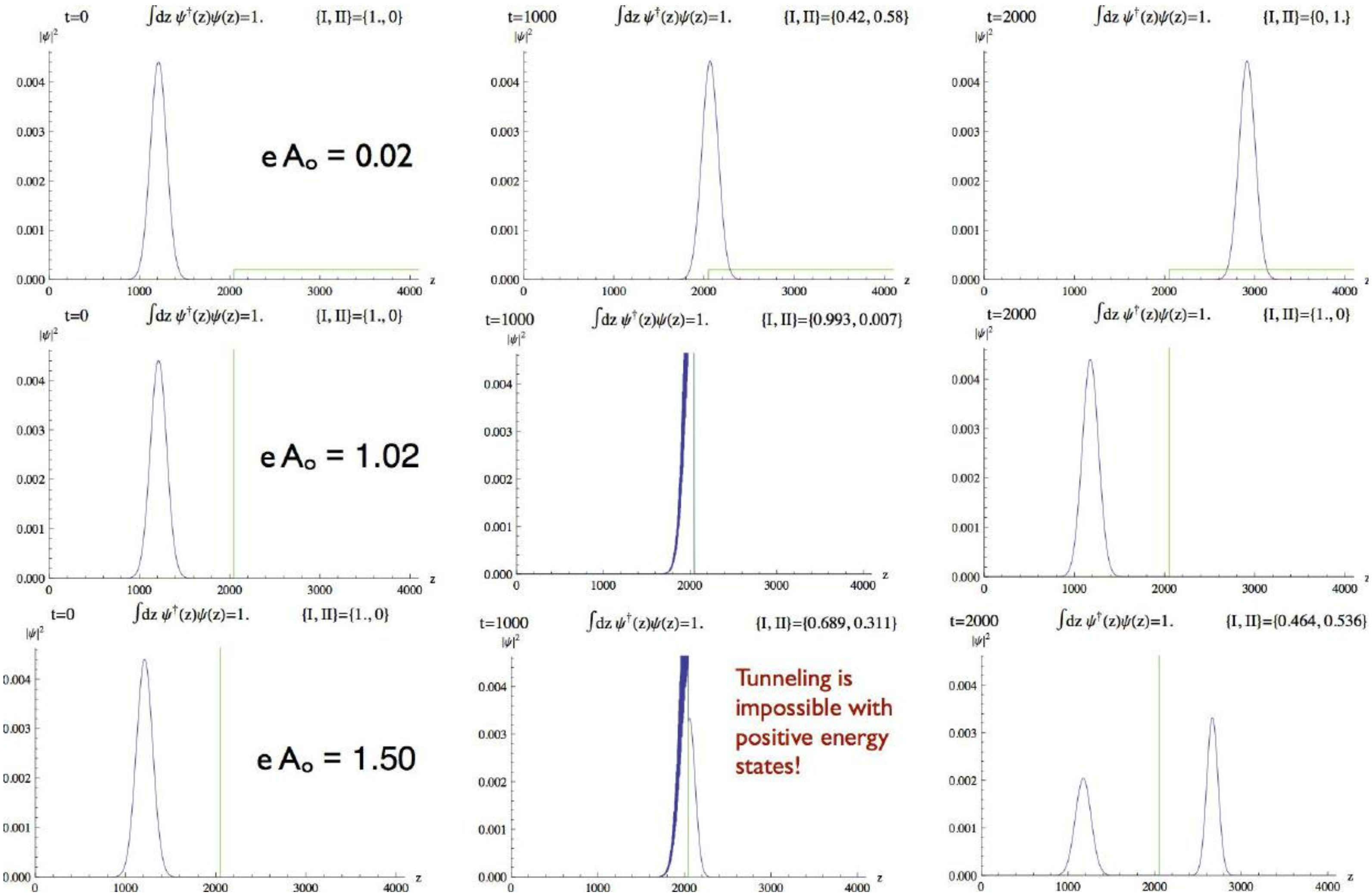
$$\begin{pmatrix} -\frac{mc}{\hbar} & i\sigma \cdot (\partial - i\frac{eA}{\hbar c}) \\ i\bar{\sigma} \cdot (\partial - i\frac{eA}{\hbar c}) & -\frac{mc}{\hbar} \end{pmatrix} \begin{pmatrix} \psi_{\mathrm{L}} \\ \psi_{\mathrm{R}} \end{pmatrix} = 0 \qquad \begin{pmatrix} -\frac{1}{\lambda_L} & i\mathbf{1} \otimes \sigma \cdot \partial \\ i\mathbf{1} \otimes \bar{\sigma} \cdot \partial & -\frac{1}{\lambda_L} \end{pmatrix} \begin{pmatrix} \mathcal{A} \\ \tilde{\mathcal{A}} \end{pmatrix} = 0$$

- London mass $m_L = \hbar/(\lambda_L c)$ breaks the chiral symmetry
- The coupled 4-spinor equations are equivalent to Maxwell-London equations [23]

$$\begin{pmatrix} -\frac{1}{\lambda_L} & i\mathbf{1} \otimes \sigma \cdot \partial \\ i\mathbf{1} \otimes \bar{\sigma} \cdot \partial & -\frac{1}{\lambda_L} \end{pmatrix} \begin{pmatrix} \mathcal{A} \\ \tilde{\mathcal{A}} \end{pmatrix} \Longleftrightarrow \begin{array}{rcl} \nabla \cdot \boldsymbol{E} & = & -A_0/\lambda_L^2 \\ \nabla \cdot \boldsymbol{B} & = & 0 \\ \nabla \times \boldsymbol{B} & = & \partial_0 \boldsymbol{E} - \boldsymbol{A}/\lambda_L^2 \\ \nabla \times \boldsymbol{E} & = & -\partial_0 \boldsymbol{B} \end{array}$$

Quantum simulations in 1+1 dimensions

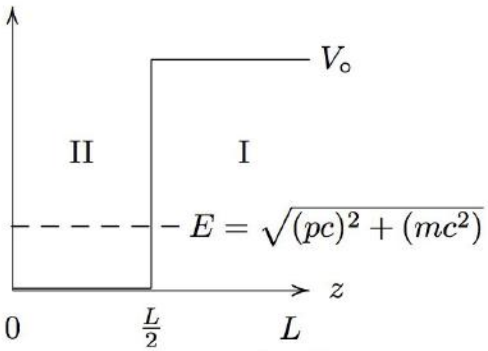
Scattering off step barrier: $i\hbar\left(\partial_t + \frac{ieA_0}{\hbar}\right)\psi = -i\hbar c\sigma_x\partial_z\psi + mc^2\sigma_z\psi$

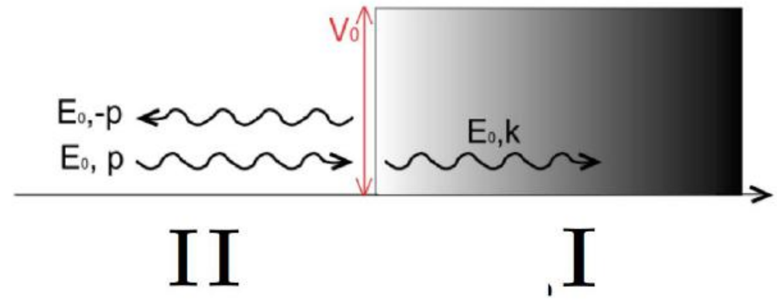


Quantum simulations in 1+1 dimensions

Scattering off step barrier: $i\hbar\left(\partial_t + \frac{ieA_0}{\hbar}\right)\psi = -i\hbar c\sigma_x\partial_z\psi + mc^2\sigma_z\psi$

$$E(z) \cong \sqrt{(pc)^2 + (mc^2)} + V_\circ\,\Theta(z - \tfrac{L}{2})$$



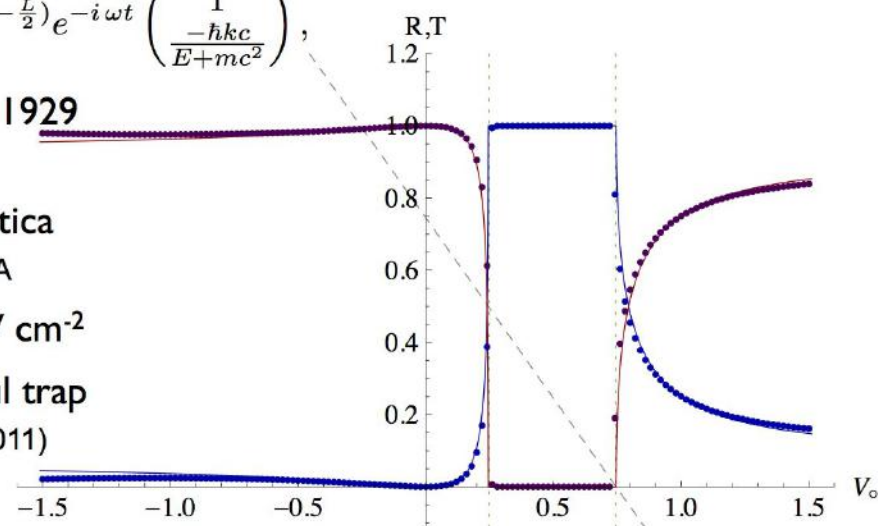


- **Solution:**

$$\psi^{\mathrm{I}}_{k'}(t,z) = A\,e^{i\,\mathrm{sgn}(E-V_\circ+m)k'(z-\frac{L}{2})}e^{-i\,\mathrm{sgn}(E-V_\circ+m)\omega' t}\begin{pmatrix}1\\ \mathrm{sgn}(E-V_\circ+m)\frac{\hbar k' c}{E-V_\circ+mc^2}\end{pmatrix}$$

$$\psi^{\mathrm{II}}_{k}(z) = e^{ik(z-\frac{L}{2})}e^{-i\omega t}\begin{pmatrix}1\\ \frac{\hbar kc}{E+mc^2}\end{pmatrix} - B\,e^{-ik(z-\frac{L}{2})}e^{-i\,\omega t}\begin{pmatrix}1\\ \frac{-\hbar kc}{E+mc^2}\end{pmatrix},$$

- Paradox discovered by physicist Oskar Klein in 1929 for the case when $V_\circ > 2\,mc^2$
- Unitary quantum algorithm run in gridMathematica
  - Recent quantum lattice gas algorithm submitted to PRA
- Electron in an intense laser field, say $I \sim 10^{29}\,\mathrm{W\,cm^{-2}}$
- Quantum Sim.: Trapped $^{40}Ca^{+}$ ion in a linear Paul trap
  - Nature Letter, **463** (2010) 68 & PRL **106**, 060503 (2011)



Quantum computing generalization of U(1) gauge theory

Add a new nonlinear interaction to a flat U(1) gauge theory (e.g. QED)

- Consider a ψ-dependent metric tensor field: $g^{\mu\nu} = \eta^{\mu\nu} + i\frac{m_\bigcirc c\ell}{\hbar}\frac{\overline{\psi}[\gamma^\mu,\gamma^\nu]\psi}{\rho_\bigcirc}$
- Gauge invariant Lagrangian density for a fermion in a self-induced curved space manifold

$$\mathcal{L} = i\hbar c g^{\mu\nu}\overline{\psi}\gamma_\mu\left(\partial_\nu + \frac{ieA_\nu}{\hbar c}\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- Euler-Lagrange equations $\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right) - \frac{\partial\mathcal{L}}{\partial\psi} = 0$ and $\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\nu)}\right) - \frac{\partial\mathcal{L}}{\partial A_\nu} = 0$ obtained by varying the action with respect to ψ and A^μ now give

$$i\hbar c g^{\mu\nu}\gamma_\mu\left(\partial_\nu + \frac{ieA_\nu}{\hbar c}\right)\psi + i\hbar c\gamma^\nu\Gamma_\nu\psi = i\frac{e^2\ell}{\hbar c}(-\lambda_\text{L}^2 e\overline{\psi}\gamma_\alpha\psi)A_\beta[\gamma^\alpha,\gamma^\beta]\psi$$

$$g^{\mu\nu}e\overline{\psi}\gamma_\mu\psi = \partial_\mu F^{\mu\nu} \qquad \Rightarrow \qquad \partial_\nu(g^{\mu\nu}e\overline{\psi}\gamma_\mu\psi) = 0$$

- Curvature correction $\Gamma_\nu = \frac{m_\bigcirc c}{2\hbar\rho_\bigcirc}\gamma_\nu S_{\alpha\beta}\psi\overline{\psi}J^{\alpha\beta}$, where $S_{\alpha\beta} = \frac{[\gamma_\alpha,\gamma_\beta]}{4}$ (spinor representation of the Lorentz group) and $J^{\alpha\beta} = i(\ell\gamma^\alpha\partial^\beta - \ell\gamma^\beta\partial^\alpha)$ (4-vector representation of the torsion)

arXiv:1612.09291 [quant-ph] (2016) [24]

Acknowledgements

I would like to extend special thanks to collaborators:

- Professor Bruce Boghosian, Mathematics Department, Tufts University
- Professor David Cory, Nuclear Engineering Department, Massachusetts Institute of Technology
- Dr. Norman Margolus, CSAIL, Massachusetts Institute of Technology
- Professor Terry Orlando, Research Laboratory of Electronics, Massachusetts Institute of Technology
- Professor Min Soe, Department of Mathematics and Physical Sciences, Rogers State University
- Professor Xerxes Tata, Department of Physics and Astronomy, University of Hawaii at Manoa
- Professor George Vahala, Physics Department, College of William and Mary
- Professor Linda Vahala, Department of Electrical & Computer Engineering, Old Dominion University
- Dr. Sean Ziegeler, High Performance Computing Modernization Program, Mississippi State University

I would like to thanks my research assistants:

- Jasper Taylor, Steven Smith & Ryan Tobin

I would like to thanks several AFOSR Program Managers and Directors:

- Dr. Jean-Luc Cambier
- Dr. Robert Bonneau
- Dr. Tatjana Curic
- Dr. Fariba Faroo
- Dr. Bill Hilbun
- Dr. Clifford Rhoades
- Dr. Bill Canfield
- Dr. Marc Jacobs

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